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Cognitive Mechanisms in Number Processing and Calculation: Evidence from Dyscalculia

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This article presents a framework for the cognitive analysis of number processing and calculation. Within this framework the primary objective is the development of a model that is sufficiently detailed to serve as a basis for explaining the number-processing/calculation performance of both normal and cognitively impaired subjects. First a general model of the cognitive mechanisms for number processing and calculation is outlined. It is shown that patterns of impairments observed in brain-damaged patients support the major assumptions of the model and that the model provides a theoretically motivated framework for interpreting the deficits. A single case is then discussed in some detail, to demonstrate that through detailed analyses of impaired performance the preliminary model can be elaborated to specify not only the general architecture of the number-processing and calculation systems, but also the inner workings of specific components and the consequences of damage to these components. The article concludes with a discussion of several general issues arising from the presented arguments.

The study of normal cognition and the study of cognitive deficits resulting from brain damage have traditionally been considered separate endeavors. However, the separation is largely artificial, because the concerns of the

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two types of research overlap to a considerable extent. The process of interpreting and classifying cognitive deficits necessarily involves assumptions about normal cognitive processing—impaired performance is interpreted as reflecting the functioning of a cognitive system in which one or more components have been damaged. Hence, a careful consideration of the nature of the normal system is critical if adequate characterizations of deficits are to be achieved. Interpretation of deficits is not, however, a simple matter of applying to the deficits a preexisting model of the normal cognitive system. Although the interpretation process may begin with at least some general assumptions about the structure of the normal system, the process—if carried out in detail—almost always forces substantial elaboration and perhaps even reformulation of these assumptions. Hence, the analysis of cognitive deficits can place strong constraints on a model of the normal system. In particular, we can require of a model of a cognitive system that it be possible to specify, for each observed pattern of impaired performance, a way of "lesioning" the system that would result in just that pattern of performance. Viewed in this way, impaired performance constitutes an integral part of the data base used to infer the structure of a cognitive system. In fact, we suggest that data from brain-damaged patients should be accorded the same status as data from normal subjects in the development of cognitive models. [See Caramazza (1984) and Shallice (1979) for more detailed discussions of the assumptions underlying the use of data from brain-damaged patients to inform the development of models of normal cognitive processing.]

In this article we examine deficits in number comprehension, number production, and calculation, considering what these deficits can tell us about the structure of the normal number-processing/calculation system, and how the deficits can be characterized in terms of damage to components of this system. We first outline a general model of the cognitive mechanisms for number use and calculation. We show that patterns of number-processing and calculation impairments observed in brain-damaged patients support the major assumptions of the model and that the model provides a theoretically motivated framework for interpreting the deficits. We then discuss a single case in some detail, to demonstrate that through detailed analyses of impaired performance our preliminary model can be elaborated to specify not only the general architecture of the number-processing/calculation system, but also the inner workings of the various components and the consequences of damage to these components. We conclude with a discussion of several general issues arising from our arguments.

COGNITIVE MECHANISMS IN NUMBER PROCESSING AND CALCULATION

The ability to understand and produce numbers may be differentiated from the ability to calculate (e.g., Cohn, 1961; Grewel, 1952, 1969;

Henschen, 1919; Hecaen, Angelergues, & Houillier, 1961). Hence, our model of the cognitive systems implicated in the use of numbers draws a basic distinction between the *number-processing system* and the *calculation system*. The number-processing system comprises the mechanisms for comprehending and producing numbers, whereas the calculation system consists of the facts and procedures required specifically for carrying out calculations. Our assumptions about the overall structure of the number-processing and calculation systems are depicted in Fig. 1. In the following discussion we consider first the number-processing system and then the calculation system.

The Number-Processing System

As shown in Fig. 1, we assume that the mechanisms for number comprehension are distinct from those for number production. Figures 2 and 3 illustrate that within the comprehension and production subsystems, we distinguish the components for processing Arabic numbers (i.e., numbers in digit form, such as 435) from the components for processing verbal numbers (i.e., numbers in the form of spoken or written number words, such as *four hundred thirty-five*). Thus, for example, reading football scores in the newspaper implicates the Arabic comprehension mechanisms, whereas writing a check involves both Arabic and verbal production mechanisms.

Within the Arabic and verbal comprehension and production mechanisms we distinguish *lexical-processing* and *syntactic-processing* components. Lexical processing involves comprehension or production of the individual elements in a number (e.g., the digit 3 or the word *three*). Syntactic processing, on the other hand, involves the processing of relations among elements in order to comprehend or produce a number as a whole. For example, comprehension of the Arabic number 4759 requires lexical processing to access the meanings of the digits 4, 7, 5, and 9, and syntactic processing that uses the positions of the digits to determine that the number is made up of four thousands, seven hundreds, and so forth. Similarly, comprehension of the verbal number *four thousand seven hundred fifty-nine* requires lexical processing to interpret the individual number words, and syntactic processing that uses word order and meanings

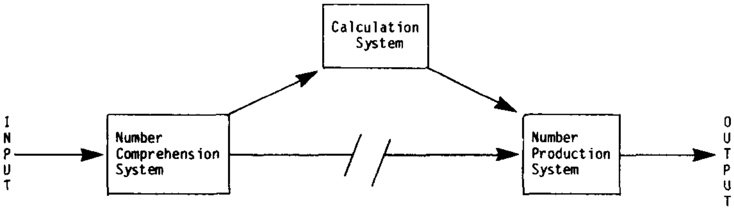

FIG. 1. Schematic representation of number-processing and calculation systems.

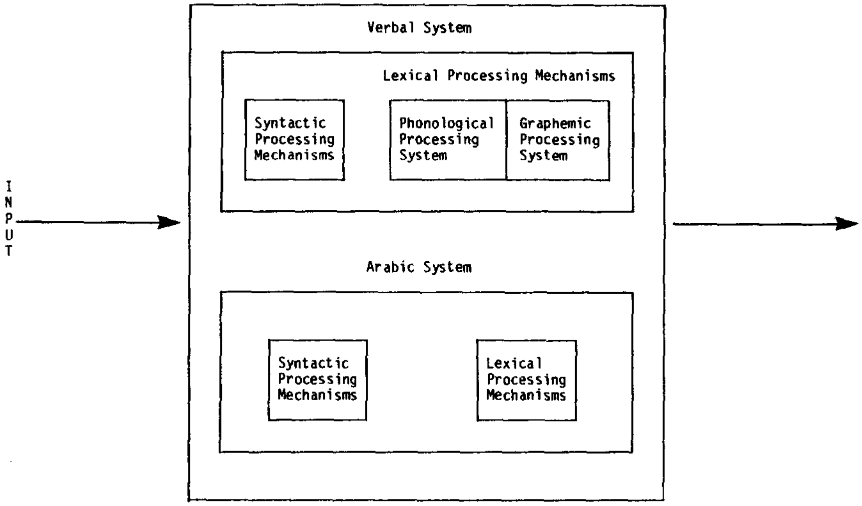

FIG. 2. Schematic representation of number-comprehension subsystem.

of words specifying powers of the number base (e.g., thousand), to construct a semantic representation of the number as a whole.

Finally, within the lexical-processing mechanisms of the verbal number system we draw a distinction between the components for producing or comprehending spoken numbers (phonological-processing components) and the components for producing or comprehending written numbers (graphemic-processing components). Thus, for example, we assume that the mechanisms for comprehending the spoken word "four" are distinct

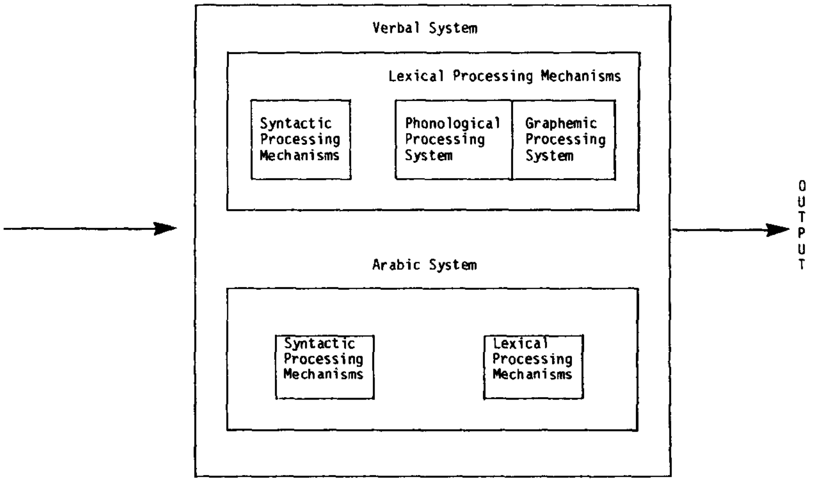

FIG. 3. Schematic representation of number-production subsystem.

from those for processing the written word *four*. We do not postulate a corresponding distinction between phonological and graphemic syntax mechanisms, because verbal number syntax is the same for spoken and written numbers. Obviously, we also do not assume separate phonological and graphemic lexical-processing components for Arabic numbers, because Arabic numbers occur only in written form.

Results from patients we have tested, and from patients described in the dyscalculia literature, strongly support this description of the general architecture of the number-processing system. In discussing these results we will not attempt to provide an exhaustive review of the dyscalculia literature,[1] or to present a comprehensive report of our studies. We simply show by considering several illustrative dissociations that our general model provides a principled basis for interpreting number-processing deficits and that interpretation of the deficits requires the distinctions we have drawn between production and comprehension mechanisms, Arabic and verbal number-processing mechanisms, and lexical and syntactic-processing mechanisms.

In drawing conclusions from a patient's performance on a task, we assume that impaired performance reflects damage to a cognitive system that was, prior to the damage, capable of performing the task successfully. For most of the data we discuss, this assumption is easily justified. In the first place, the tasks we consider (e.g., reading an Arabic number such as 4235 aloud, performing simple arithmetic such as 234 + 26) require only very basic number processing and calculation skills. Consistent with this assessment, control subjects comparable in age and education to our patients performed virtually without error on all of the tasks. Further, in many instances information is available to show that a patient was capable premorbidly of performing a task (e.g, the patient's job required extensive number processing and calculation). Nevertheless, it is important to mention that in some instances the possibility of a premorbid difficulty with a task cannot be ruled out entirely.

Comprehension/production dissociations. The available data provide considerable evidence in favor of the distinction between number-comprehension and number-production mechanisms. Benson and Denckla (1969) describe a patient presenting with intact comprehension but impaired production of verbal and Arabic numbers. For arithmetic problems (e.g.,

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[1] Our discussion of previous work focuses on single case reports because these provide the detailed analyses of patients' performance needed to address the theoretical issues of interest here. Of course, the literature on cognitive disorders also includes many group studies concerning dyscalculia (e.g., Collignon, Leclercq, & Mahy, 1977; Dahmen, Hartje, Bussing, & Sturm, 1982; Grafman, Passafiume, Faglioni, & Boller, 1982; Hecaen, Angelergues, & Houillier, 1961). However, these studies have typically considered different sorts of issues from those we address in this article. For recent reviews discussing the group studies see Boller and Grafman (1983) and Levin (1979).

4 + 5, or 372 + 69) presented visually in Arabic form or aurally in verbal form, the patient could consistently choose the correct answer from a set of possible answers. This result implies an ability to comprehend the Arabic numbers in the written problems and the verbal numbers in the aurally presented problems. Hence, at least within the range of numbers tested, Arabic and verbal number comprehension were unimpaired. However, when the patient was asked to say or write the answer to a problem, he almost always answered incorrectly. Benson and Denckla report, for instance, that for the written problem 4 + 5, the patient said "eight," wrote "5", and chose "9" from a multiple-choice list. Similarly, when a verbal number (e.g., *two hundred twenty-one*) was presented aurally and the patient was asked to write the Arabic equivalent, the digits in his response were frequently incorrect (e.g., 215 written for 221); and when the patient was asked to say the verbal number corresponding to an Arabic digit or to a number of dots, his responses were usually wrong. The patient's excellent performance on the multiple-choice arithmetic problems strongly suggests that the errors on tasks requiring written or spoken responses reflect an impairment in producing the responses, and not a difficulty in comprehending the stimuli. Hence, the patient shows a dissociation between number comprehension (intact) and number production (impaired), supporting the assumption that the cognitive mechanisms for number comprehension are distinct from those for number production.

The patient described some years ago by Singer and Low (1933) shows a similar dissociation in the processing of Arabic numbers. This patient's comprehension of Arabic numbers was intact, as indicated by his ability to judge which of two Arabic numbers was larger (e.g., 305 vs. 503), and to select a number specified verbally from a column of Arabic numbers (e.g., which number is *seven hundred twenty-five*?). However, the patient was unable to write to dictation numbers above 100. For example, when asked to write *two hundred forty-two*, the patient produced 20042. The patient's ability to select a dictated number from a list strongly suggests that his errors in writing numbers reflected a deficit in the production of the Arabic responses, and not a problem in understanding the dictated verbal stimuli. Hence, like the patient described by Benson and Denckla, Singer and Low's patient evidenced a number-production deficit in the presence of intact number comprehension.

Arabic/verbal dissociations. Evidence can also be adduced in support of the assumption that the processing mechanisms for Arabic numbers are distinct from those for verbal numbers. Our patient H.Y. made no errors in judging which of two Arabic numbers was larger (e.g., 4 vs. 3; 27,305 vs. 27,350), suggesting intact comprehension of Arabic numbers. However, H.Y. performed at chance on magnitude comparison judgments for small or large verbal numbers presented visually (e.g., *four* vs. *three;*

six thousand four hundred vs. *seven thousand nine hundred*), indicating impaired comprehension of verbal numbers. In contrast, patient K. evidenced a deficit involving Arabic but not verbal numbers. This patient performed without error in judging which of two number words was larger (e.g., *six* vs. *seven*), but showed near-chance performance (67% correct) on magnitude comparison judgments for Arabic digits (e.g., 6 vs. 7).

Berger (1926) describes two patients who represent a double dissociation of Arabic and verbal number production. One patient, when presented with simple arithmetic problems, gave correct spoken (verbal) answers while writing incorrect (Arabic) answers. For example, given 10 × 5, the patient said "fifty" but wrote "32." Another patient, in contrast, gave correct written answers but incorrect spoken responses (e.g., for 24 ÷ 6 the patient wrote "4" but said "two"). (This latter patient also showed a clear comprehension/production dissociation for verbal numbers, for he gave correct written answers to problems presented aurally, indicating that he could comprehend the verbal problems.)

Lexical/syntactic dissociations. Lexical and syntactic processing of numbers are clearly dissociable. The number-production deficit evidenced by Benson and Denckla's (1969) patient apparently involved lexical but not syntactic processing. In saying or writing numbers the patient's responses were well formed (i.e., responses like "fifteen two hundred" apparently never occurred) and of the appropriate order of magnitude, but included incorrect digits or number words. For example, when asked to write *two hundred twenty-one,* the patient produced 215, which is of the correct order of magnitude but includes incorrect digits (1 and 5). This pattern of peformance suggests a deficit in producing the individual elements of a number (i.e., individual digits such as 4, or individual number words such as *three*), but a preserved ability to assemble the (possibly incorrect) elements into a number of the appropriate syntactic form and order of magnitude. Thus, Benson and Denckla's patient shows impaired lexical processing but intact syntactic processing in the production of numbers.

In contrast, the patient described by Singer and Low (1933; see also Sittig, 1919) presented with a pattern of performance suggesting impaired syntactic processing in the presence of normal lexical processing. When verbal numbers (e.g., *two hundred forty-two*) were presented aurally and the patient was asked to write the Arabic equivalents, he produced responses in which the individual digits were correct but the order of magnitude was incorrect. For example, the patient, who premorbidly held a job requiring considerable number processing and calculation, wrote *two hundred forty-two* as 20042 and *two thousand five hundred* as 2000500. The production of the correct digits implies intact lexical processing. However, the fact that the responses were usually of the wrong order of magnitude suggests an impairment in syntactic processing. More specifically, the patient's deficit apparently involved syntactic processing in the production of Arabic numbers—as noted earlier, the patient's errors in number writing apparently reflected a deficit in production of the Arabic responses, not a problem in comprehending the verbal stimuli. Thus, in producing Arabic numbers the patient was able to select the appropriate digits (lexical processing), but was unable to assemble these digits into the appropriate whole number (syntactic processing).

Our patient V.O. presented with a very similar pattern of performance. Like Singer and Low's patient, V.O. showed intact comprehension of Arabic and verbal numbers, but made syntactic errors in writing Arabic numbers. Examples of V.O.'s responses on the number-writing task are presented in Table 1.

Benson and Denckla's patient showed, then, impaired lexical and intact syntactic processing, whereas Singer and Low's patient and our patient evidenced impaired syntactic and intact lexical processing. This pattern of dissociations strongly supports our distinction between lexical and syntactic number-processing mechanisms.[2]

Phonological/graphemic dissociations. Consider finally the distinction between phonological and graphemic lexical-processing mechanisms. This distinction is supported by data showing that the ability to process the elements of spoken and written verbal numbers can be disrupted independently. For example, patient H.Y. performed at chance in judging which of two written number words was larger (e.g., six vs. five), but showed perfect performance on the corresponding task involving spoken number words. (Note that the deficit for written numbers was not due to a peripheral visual perceptual problem, because H.Y. evidenced error-free performance in judging which of two single-digit or multidigit Arabic numbers was larger.)

In this discussion we have not demonstrated all possible dissociations among the components specified in our model of the number-processing system. However, the dissociations we have described clearly motivate our assumptions about the general architecture of the number-processing system. Further, the data suggest that number-processing impairments can fruitfully be characterized in terms of whether they involve number comprehension or number production; whether they involve Arabic or verbal numbers; whether they involve lexical or syntactic processing; and, in the case of lexical processing of verbal numbers, whether processing of written or spoken numbers is impaired.

We turn now to the general architecture of the calculation system.

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[2] Deloche and Seron (1982a, 1982b) have also described lexical and syntactic errors made by patients in number-writing tasks. However, the dissociation of lexical and syntactic processing is not clear from Deloche and Seron's reports, because their procedure of pooling errors across subjects makes it difficult to determine whether some subjects made only lexical or only syntactic errors.

TABLE 1
EXAMPLES OF THE PERFORMANCE OF PATIENT V.O. ON A NUMBER-WRITING TASK

Stimulus	Response
Eight	8
Five	5
Zero	0
Six	6
Forty	40
Two hundred thirty-seven	237
Seven thousand forty	700040
Three thousand six hundred fifty-nine	300060059
Forty-seven thousand	47000
Four hundred	400
Forty thousand seven	400007
Five thousand seventeen	500017
Four hundred thirty-seven thousand	40037000

The Calculation System

Any calculation task requires some sort of number-production and/or comprehension abilities. Hence, damage to a component of the number-processing system should lead to deficits on calculation tasks requiring that component of processing. For example, a lexical deficit in the comprehension of Arabic numbers would presumably lead to impaired performance on arithmetic problems presented in Arabic form (but not on problems presented aurally). However, calculation may also be impaired by damage to components of processing that are concerned specifically with calculation. In this section we discuss these components, which we refer to collectively as the calculation system.

Our model of the calculation system specifies three major components, as shown in Fig. 4. Specifically, we assume that calculation requires, in addition to number-processing mechanisms, cognitive mechanisms for (1) processing of operational symbols (e.g., +) or words (e.g., plus) that identify the operation to be performed; (2) retrieval of basic arithmetic facts (i.e., table facts such as $6 \times 7 = 42$); and (3) execution of calculation procedures (e.g., to add two multidigit numbers, start at the rightmost column, retrieve the sum of the digits in the column, write the ones digit of the sum at the bottom of the column, set the carry flag if the sum is greater than nine, shift one column left, and so forth). As in the case of the number-processing system, patterns of deficits evidenced by our patients and by patients described in the literature on dyscalculia support these distinctions among system components.

In the first place, the ability to process operational symbols or words can be disrupted selectively (e.g., Ferro & Botelho, 1980; Grewel, 1952, 1969). Ferro and Botelho describe a patient who was unable to comprehend

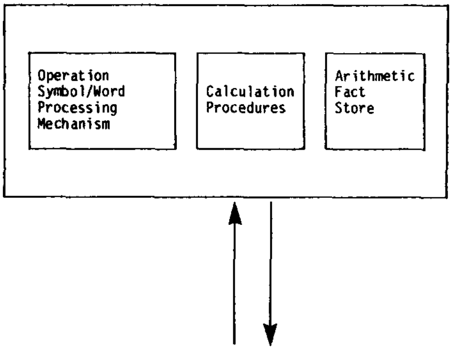

FIG. 4. Schematic representation of calculation system.

the operation symbols, so that when presented with a written arithmetic problem she often performed (correctly) the wrong operation (e.g., 3 × 5 = 8; 6 + 3 = 18; 35 + 24 = 840). However, when problems were presented verbally, the patient showed excellent performance. The normal performance on verbal problems and the fact that on written problems the wrong operations were performed correctly (e.g., for 35 + 24, the two numbers were multiplied correctly) imply that the patient was unimpaired in retrieval of arithmetic facts and execution of calculation procedures. This pattern of performance suggests, then, that processing of the operation symbols is independent of arithmetic fact retrieval and execution of calculation procedures.

It is interesting to note that this patient's comprehension of Arabic numbers was clearly intact (as indicated by the correct performance of the wrong operation on written arithmetic problems, and by other data). Hence, the patient shows a dissociation between comprehension of the digit symbols (intact) and comprehension of the operation symbols (impaired).

Fact/procedure dissociations. The two major components of the calculation system—the arithmetic fact and calculation procedure components—can also be disrupted selectively. Some patients show intact fact retrieval in the presence of impaired ability to execute calculation procedures, whereas other patients show impaired fact retrieval in the presence of intact ability to execute calculation procedures.

Warrington (1982) has described in detail a patient who presented with difficulties in accessing arithmetic facts. D.R.C, a physician, showed normal number comprehension and production. However, he evidenced a clear impairment in retrieval of basic arithmetic facts, as indicated by errors (e.g., 5 + 7 = 13) and abnormally slow response times. Interestingly, the patient was able to give reasonable definitions of the four basic arithmetic operations.

Several other researchers have also reported arithmetic fact retrieval deficits (e.g., Grewel, 1952, 1969; Cohn, 1961). However, these reports typically have not ruled out number-processing problems as a possible explanation for the poor arithmetic performance, and have not demonstrated dissociations of fact retrieval and execution of calculation procedures.

A clear case of selective disruption of arithmetic fact retrieval is provided by our patient M.W. This patient, who has a doctorate in social work, was able to execute the calculation procedures flawlessly, and showed a clear understanding of the arithmetic operations. However, he was severely impaired in the retrieval of arithmetic facts, especially in multiplication. In multiplying single-digit numbers, M.W. frequently produced an incorrect response (e.g., 6 × 7 = 48) or was unable to retrieve an answer. In the latter instance he often used facts he could retrieve to calculate the answer. Hence, when M.W. was unable to remember 7 × 7, he calculated the product as 70 (7 × 10) − 21 (7 × 3). Thus, M.W. clearly understood the multiplication operation quite well. On multidigit multiplication probems M.W. consistently executed the multiplication procedure correctly but again evidenced difficulty in arithmetic fact retrieval. For example, as shown in Fig. 5, M.W. multiplied 443 × 92 correctly, except that he arrived at 24 when multiplying 9 × 3.

We have studied M.W.'s multiplication fact retrieval in some detail, by presenting a large number of problems involving the numbers 1 through 10 (e.g., 6 × 7, 4 × 5). For both written and oral presentation and response modes, M.W.'s pattern of performance was the same—he consistently had difficulty in retrieving certain facts. Table 2 shows the percentage of errors for each multiplication fact for the numbers 1 through 10 (a dash indicates perfect performance). The row and column labels in the table represent operands in a problem. Thus, for example, the

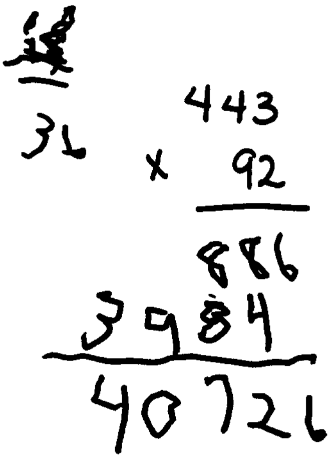

FIG. 5. Example of patient M.W.'s multiplication performance showing intact calculation procedure and impaired fact retrieval.

TABLE 2
PERCENTAGE OF ERRORS ON BASIC MULTIPLICATION FACTS FOR PATIENT M.W.

×	1	2	3	4	5	6	7	8	9	10
1	—*	—	—	5	—	—	6	7	—	—
2		9	7	—	—	10	8	—	13	—
3			7	—	—	11	8	10	31	—
4				12	—	8	20	27	5	—
5					—	—	40	10	36	—
6						—	19	7	48	—
7							13	59	3	—
8								62	23	—
9									44	—
10										—

* Dash indicates perfect performance.

entry at the intersection of the fourth row and the sixth column gives the percentage of errors for the problems 4 × 6 and 6 × 4.

The data clearly indicate that M.W.'s difficulty with multiplication cannot be attributed to number comprehension or production problems. Although this point could be made by considering performance on number comprehension and production tasks, it is worthwhile to demonstrate how number-processing problems can be ruled out by considering only M.W.'s performance on multiplication tasks. As shown in Table 2, M.W.'s performance on ones and tens problems (e.g., 5 × 1, 10 × 8) was virtually perfect. This result indicates that M.W. had no difficulty in comprehending the numbers in the problems, and hence that his errors cannot be attributed to a number-comprehension impairment.

Two sorts of results indicate that M.W.'s arithmetic deficit was not due to a number-production impairment. First, M.W. made errors not only when he had to produce the answer to a problem, but also in verification tasks in which he did not have to produce an answer, but only had to indicate whether a given product was correct or incorrect. (For example, M. W. indicated that 7 × 8 = 49 was correct, and that 6 × 9 = 54 was incorrect.) The second result arguing against a number-production deficit interpretation of M.W.'s arithmetic difficulty concerns the nature of the errors he made in producing responses to multiplication problems. Of the 106 errors made by M.W., 97 were "within-table" errors. That is, the incorrect responses were almost always products in the 1 to 10 times table. For example, M.W.'s errors included the numbers 56 and 48, but not 59 or 47. Furthermore, 85% of M.W.'s errors were products of one of the multiplicands in the problem (e.g., 6 × 8 = 56, where 56 is a product of 8). This pattern of errors cannot be explained by assuming that M.W. retrieves the correct answer to a problem, but then makes an error in saying or writing that answer. Errors occurring as a result of a number-production deficit would not be limited to numbers in the multiplication table or, more specifically, to products of one of the operands in a problem. A final result arguing against a number-processing defict interpretation of M.W.'s performance is that, as mentioned above, M.W. often clearly revealed that he could not retrieve the answer to a problem, by calculating the answer from facts he could retrieve (as when 7 × 7 was computed as 7 × 10 − 7 × 3).

The data also reveal some interesting points about the nature of M.W.'s fact retrieval deficit. First, it is clear that the degree of retrieval difficulty is not the same for all of the facts—the deficit is clearly more severe for some specific facts than for others. For example, M.W. had great difficulty with 8 × 8 and 8 × 7, but not with 8 × 6 or 9 × 7. Second, the finding that M.W.'s erroneous responses were usually products of one of the multiplicands suggests that to a large extent his deficit involves accessing the wrong fact in the arithmetic fact store. In other words, when M.W. attempts to retrieve the product of 7 and 8 he may access the product of 6 and 8 instead.

M.W., then, shows a selective impairment of the arithmetic fact system. The pattern of spared and impaired functions clearly indicates that he can execute calculation procedures normally, but has difficulty in retrieving arithmetic facts.

Contrasting with patients showing selective arithmetic fact retrieval deficits are patients with selective impairments in the execution of calculation procedures. For example, patient 1373 in the Vietnam Head Injury Study presented with a deficit involving the multiplication procedure. As shown in Fig. 6, he consistently failed to shift the second row of intermediate products one column to the left. However, the patient invariably retrieved the correct multiplication facts. Note too that the patient added the intermediate products correctly. This sort of deficit has also been described by Grewel (1969).

Calculation procedure deficits take many different forms, reflecting disruption of different components of the calculation procedure system.

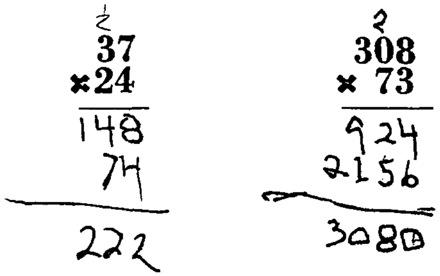

FIG. 6. Examples of patient 1373's multiplication performance showing failure to shift the intermediate product in the second row.

Some patients have difficulty with the carry and/or borrow operations. For example, as shown in Fig. 7, our patient V.O. failed to carry consistently when adding. A more complex and interesting form of carry operation deficit involves the misordering of the carry step in relation to other steps in the calculation procedure. Thus, as shown in Fig. 8, our patient D.L. carried in multiplication by adding the carry digit to the next multiplicand before multiplying, rather than to the intermediate product after multiplying. For example, in multiplying 73 × 5, D.L. said, "three times five is fifteen, carry the one; one and seven is eight, and eight times five is forty." Prior to his head injury, D.L. had successfully completed several accounting courses.

Another common procedural deficit involves the failure to separate intermediate sums or products into ones and tens digits, so that the tens digit can be carried. Instead, the patient writes down the complete intermediate sum or product. Figure 9 shows examples of this type of deficit in addition and multiplication. Head (1926) and Grewel (1969) have also reported deficits of this sort.

A particularly interesting type of deficit involves the apparent confusion of component steps for one operation with the steps of another operation. As shown in the left half of Fig. 10, one of our patients (W.W.) apparently used components of the multiplication procedure in an addition problem: the patient added 45 and 8 by first adding 8 and 5, and then 8 and 4. Note that the patient also wrote down the complete intermediate sum, rather than carrying the tens digit. Another patient (H.Y.) apparently applied the addition procedure to a multiplication problem (although he retrieved multiplication facts). This patient, as shown in the right half of Fig. 10, multiplied 58 × 69 by multiplying 8 × 9 and then 6 × 5.

The calculation procedures may also be more drastically disrupted, as illustrated by the two examples in Fig. 11. In the example on the left, the patient obtained the answer 28 for the problem 68 + 59, saying while

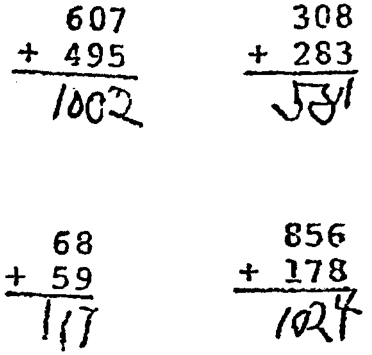

FIG. 7. Examples of patient V.O.'s addition performance showing failure to carry consistently.

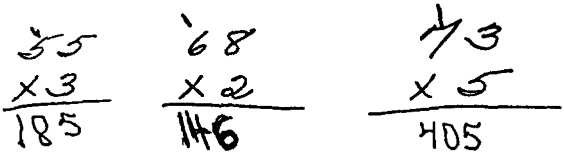

FIG. 8. Examples of patient D. L.'s multiplication performance showing inappropriate carry procedure.

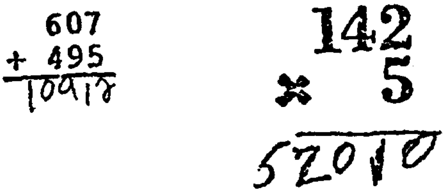

FIG. 9. Examples of two patients' inappropriate treatment of intermediate sums and products.

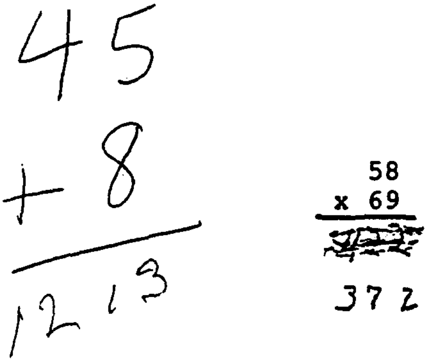

FIG. 10. Examples of performance by patients W.W. and H.Y. showing apparent confusion of component steps of one operation with steps of another operation.

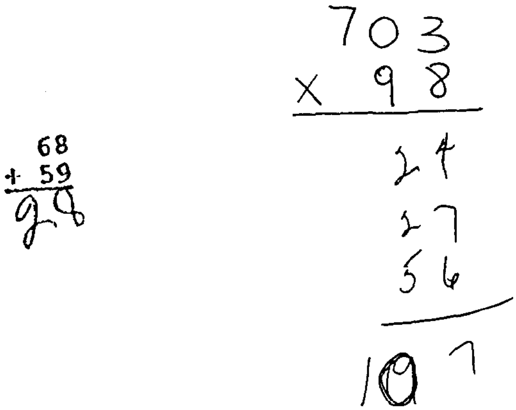

FIG. 11. Examples of two patients' performance showing drastic disruption of calculation procedures.

he worked the problem, "8 + 9 is 17, 6 + 5 is 11, 17 + 11 is 28." In the example on the right, the patient is grossly impaired in organizing the steps of the multiplication procedure, and in arranging the intermediate products.

Dissociations of arithmetic operations. An interesting question we can ask for both arithmetic fact retrieval and execution of calculation procedures is whether independent mechanisms underlie the different arithmetic operations (i.e., addition, subtraction, multiplication, and division). In the realm of procedural dyscalculia our results and those reported by other researchers (e.g., Berger, 1926; Cohn, 1961) clearly show that some operations (e.g., division) may be impaired while others (e.g., addition) remain intact. Patients with clear multiplication procedure deficits, for example, often add the (incorrect) intermediate products flawlessly, as illustrated in Figs. 6 and 11. Further, we have recently observed a dissociation in which multiplication was impaired while division remained intact. As illustrated in Fig. 12, patient 1373 from the Vietnam Head Injury Study exhibited a clear multiplication procedure deficit, but performed long division correctly. Although more evidence is clearly needed, this finding suggests that the procedures for each of the basic operations may be represented autonomously in the calculation procedure system.

In the realm of arithmetic fact retrieval deficits, our results indicate that multiplication fact retrieval may be impaired while the ability to retrieve addition and subtraction facts remains intact. Also, Berger (1926) has described several cases in which it appears that fact retrieval was intact for addition and multiplication, but impaired for subtraction and division. Hence, there may be separate arithmetic fact systems for each of the basic operations.

The general model we have presented provides a framework for thinking about normal and impaired number processing and calculation, and is

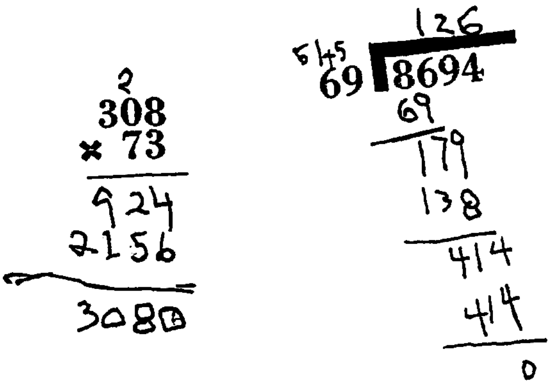

FIG. 12. Examples of patient 1373's performance showing intact performance in division and impaired performance in multiplication.

supported by the dissociations we have discussed. However, while the model is considerably more explicit than any developed in previous work on dyscalculia, it is by no means fully specified–the structure and functioning of the postulated components are not described in sufficient detail to explain how given a particular input (e.g., 3040 in a number-reading task) the system produces the desired output (e.g., "three thousand forty"). What, for example, is involved in lexical processing during the comprehension of an Arabic number, and how is syntactic processing in verbal number production carried out? Ultimately, the utility of our framework will depend upon whether satisfactory answers to these sorts of questions can be developed. To illustrate how the analysis of patterns of impaired performance can provide a basis for elaborating the general model we have proposed, we describe in the next section a case study of a patient—R.R.—who displays a striking deficit when he is asked to read Arabic numbers aloud.

A CASE OF IMPAIRED LEXICAL PROCESSING IN VERBAL NUMBER PRODUCTION

Table 3 presents several examples of patient R.R.'s performance in reading numbers aloud. As the examples illustrate, the individual number words (e.g., *five, seventy*) in R.R.'s responses were usually wrong. However, the responses were syntactically well formed, and were almost always of the correct order of magnitude. For example, stimuli in the hundreds usually elicited responses in the hundreds, thousands stimuli elicited thousands responses, and so forth. Thus, when the stimulus 37,000 was presented, R.R. said "fifty-five thousand." This pattern of performance, which suggests impaired lexical processing in the presence of largely intact syntactic processing, was highly stable over a period of several weeks during which we asked R.R. to read over 1200 numbers ranging in magnitude from one to six digits.

TABLE 3
EXAMPLES OF THE PERFORMANCE OF PATIENT R.R. ON A NUMBER-READING TASK

Stimulus	Response
37,000	Fifty-five thousand
40	Fifty
130,000	One hundred thirty-three thousand
2	One
4253	Two thousand five hundred twenty-five
27,360	Twenty-five thousand five hundred thirty-two
426	Four hundred thirty-five
7	Four
62	Thirty-four

R.R.'s errors cannot be attributed to an inability to comprehend the digits in the Arabic stimuli. He responded rapidly and without error when asked to indicate which of two single-digit or multidigit Arabic numbers was larger (e.g., 4 vs. 3; 45,678 vs. 46,561). Also, R.R. showed excellent performance when an Arabic digit was presented and he was asked to select the corresponding number of tokens from a pile. These results strongly suggest that R.R.'s lexical deficit is not in comprehending the Arabic stimuli, but in producing the verbal responses.

An analysis of R.R.'s performance leads to specific conclusions about the structure of the verbal number-production system, and the nature of R.R.'s deficit. To make clear the basis for these conclusions, we must describe R.R.'s performance in some detail.

R.R. produced the correct number word (e.g., *sixty*) for a digit in a stimulus (e.g., the 6 in 467) only about 38% of the time. This percentage was independent of the size of the stimulus number and of the position of a digit in the number. Thus, for example, the probability of a correct response to a single-digit number was no higher than the probability of responding correctly to a particular digit in a six-digit number. Further, the digit in the ones position of a number (e.g., the 2 in 6592) was no more likely to be produced correctly than the digit in the thousands position (e.g., the 2 in 42,591).

Another interesting feature of R.R.'s responses is that incorrect number-word responses showed virtually no tendency to be close in magnitude to correct responses. Thus, for example, given the stimulus 3, R.R. was no more likely to say "four" than "seven."

However, examination of R.R.'s responses reveals that his errors were constrained in an interesting way. R.R.'s responses were of the correct order of magnitude nearly 90% of the time: single-digit stimuli elicited responses in the range zero to nine, two-digit stimuli elicited responses in the range ten to ninety-nine, three-digit stimuli elicited responses in the hundreds, and so forth.

Additional structure was present in R.R.'s responses to two-digit numbers. In particular, stimuli in the teens (10–19) usually elicited responses in the teens. This was true both for teen numbers in isolation (e.g., stimulus 17, response "thirteen"), and for teens embedded in larger numbers (e.g., stimulus 15,200, response "seventeen thousand three hundred"). However, there was no tendency for stimuli in the twenties (alone or embedded in larger numbers) to elicit responses in the twenties, for thirties stimuli to elicit thirties responses, or so forth. Stimuli in the 20–99 range elicited responses throughout that range (e.g., stimulus 21, response "sixty-seven"). For example, whereas the probability of a teens response to a teens stimulus was .87, the probability of a twenties response to a twenties stimulus was only .40.

The tendency for stimuli in the 10–19 range to elicit responses in that range did not occur because the digit "1" was somehow processed more accurately than other digits. The proportion of teen responses to teen stimuli was much higher than the proportion of correct responses to a 1 in a nonteen position (e.g., the 1 in 21 or 153): .87 vs. .56. (Note that this large difference in performance between "teen" and "nonteen" 1's provides further support for our assumption that the locus of R.R.'s deficit is *not* in the lexical processing of the Arabic stimuli.)

Thus, R.R.'s error rate is very high, but his errors are highly constrained. If the correct word is "six" R.R. may say "three," but not "fourteen" or "fifty"; if the correct response is "seventeen," R.R. may say "nineteen" but not "five" or "thirty"; and if the correct response is "eighty" R.R. may say "twenty" but not "thirteen" or "two."

This pattern of performance provides a basis for inferences about the verbal number-production system. To read a number aloud one must access stored information about the phonological forms of the words to be produced; this stored information may be referred to as the phonological number-production lexicon. The structured pattern of lexical errors in R.R.'s number reading suggests that the production lexicon is organized into three functionally distinct classes, as shown in Table 4. The ONES class contains the phonological specifications for the words one through nine, the TEENS class contains information about the words ten through nineteen, and the TENS class contains the phonological forms for the words twenty, thirty, forty, and so forth, up to ninety.

A person reading a number must select the appropriate lexical class and the appropriate item within class. We assume that R. R.'s ability to select the lexical class is intact, but that he is severely impaired in selecting the item within class. For example, in reading the number 4, R.R. accesses the ONES class, and so does not produce "sixteen" or "seventy." However, because of his impairment in selecting the item within class he may retrieve "one" or "seven" rather than "four."

TABLE 4
Number Lexical Classes in the Verbal Number-Production Lexicon

ONES	TEENS	TENS
—	Ten	—
One	Eleven	—
Two	Twelve	Twenty
Three	Thirteen	Thirty
Four	Fourteen	Forty
Five	Fifteen	Fifty
Six	Sixteen	Sixty
Seven	Seventeen	Seventy
Eight	Eighteen	Eighty
Nine	Nineteen	Ninety

Thus, the distinction among lexical classes and the interpretation of R.R.'s deficit are motivated by the fact that R.R.'s number-reading errors respect the boundaries between the postulated classes.

Our assumptions about the structure of the number-production lexicon can be placed within the context of a procedurally explicit model of the number-reading process. We assume that when a to-be-read number is presented, a number-production syntax device generates a syntactic frame on the basis of the number of digits in the number. For example, for the number 4765 the syntactic frame would take the form

$$\underset{\text{ONES}}{\underline{\qquad\quad}}\ \text{T}\ \underset{\text{ONES}}{\underline{\qquad\quad}}\ \text{H}\ \underset{\text{TENS}}{\underline{\qquad\quad}}\qquad \underset{\text{ONES}}{\underline{\qquad\quad}}\cdot$$

On the basis of lexical processing of the stimulus number, the labeled slots in the frame are filled in with representations of the individual quantities in the number. Thus, the filled frame for the number 4765 would look like

$$\underset{\text{ONES}}{\underline{\quad\text{“4”}\quad}}\ \text{T}\ \underset{\text{ONES}}{\underline{\quad\text{“7”}\quad}}\ \text{H}\ \underset{\text{TENS}}{\underline{\quad\text{“6”}\quad}}\qquad \underset{\text{ONES}}{\underline{\quad\text{“5”}\quad}}\cdot$$

Each filled slot specifies a phonological form to be retrieved from the number-production lexicon. The label (e.g., ONES) specifies the lexical class, and the quantity representation specifies the item within class. Thus, for example, the leftmost slot specifies retrieval from the ONES class, so that for this slot the phonological form /four/ will be retrieved. (We indicate the phonological form of a word by slashes enclosing the word.)

Syntactic frames also include, where appropriate, instructions to retrieve the phonological forms of words such as hundred, thousand, and million, which in the verbal number code indicate the power of ten associated with each individual quantity in the number. In our example, the T and H represent instructions for retrieval of the phonological forms /thousand/ and /hundred/.

Production of a number involves successive retrieval of the phonological forms specified by the filled syntactic frame. In the present example the process would result in the successive retrieval of the phonological forms /four/ /thousand/ /seven/ /hundred/ /sixty/ /five/.

The number-production process proceeds as described, except in certain special cases. Of particular relevance here is the case of teen numbers. We assume that when the production process encounters a "1" in a slot with a "TENS" label, a special procedure is invoked. This procedure specifies that nothing should be retrieved from the TENS class, but that the quantity representation in the next slot should be used to retrieve a phonological form from the TEENS class. Hence, for the number 4715, the string produced would be /four/ /thousand/ /seven/ /hundred/ /fifteen/.

Given this model of the verbal number-production system, we can provide a more detailed characterization of spared and impaired abilities in patient R.R. We assume that R.R. usually generates the correct syntactic frame, fills it with the correct quantity representations, and accesses the appropriate lexical classes when retrieving phonological forms. However, he is severely impaired in selecting the correct item within a class. Thus, we explain R.R.'s performance in terms of damage to an explicitly described normal system; and at the same time R.R.'s performance provides strong empirical support for assumptions about the structure of that system.

GENERAL VS. SPECIFIC DEFICITS

One matter we have not yet discussed concerns the general or specific nature of deficits. We have shown that by testing a patient with tasks chosen on the basis of a model of normal number processing and calculation, we can make inferences about what stages of processing are disrupted. Consider, for example, a patient who shows poor performance on pencil-and-paper multiplication of single-digit numbers. According to our model, pencil-and-paper multiplication involves, in addition to peripheral perceptual and motoric processing, comprehension of the numbers in the problem, comprehension of the operation sign, retrieval of multiplication facts, and production of the retrieved number. Assume that we find normal performance on tests of Arabic number comprehension and production and on tests of operation sign comprehension. Assume further that the subject's deficit on the multiplication task consists of the frequent production of incorrect responses that are multiples of one of the multiplicands in the problems (e.g., $6 \times 4 = 18$, $8 \times 7 = 64$). On the basis of these results we could infer that the patient has a deficit in the stage of processing that retrieves stored arithmetic facts.

However, once we have specified the disrupted stage(s) of processing, the question remains whether the disruption represents damage to a cognitive mechanism specific to that processing stage or instead to a mechanism that is more general in nature. For the patient with a deficit in arithmetic fact retrieval, we may ask whether the deficit involves damage to some cognitive mechanism specific to arithmetic fact retrieval or disruption of some cognitive structure or process that is implicated not only in the retrieval of arithmetic facts, but also in the retrieval of other sorts of stored information. Similarly, in the case of patient R.R. we can ask whether his lexical number-production deficit reflects damage to some mechanism specifically involved in the retrieval of the phonological forms of number words, or whether instead a more general lexical-processing mechanism (i.e., a mechanism involved in the retrieval of phonological forms for words in general) is disrupted.

In previous discussions of dyscalculia the general vs. specific issue has been raised repeatedly in regard to several different types of deficits. For example, many researchers have attributed various forms of calculation impairment to a general spatial-processing deficit (e.g., Collignon, Leclercq, & Mahy, 1977; Hecaen et al., 1961; Krapf, 1937; Luria, 1966). Similarly, it has often been assumed that some or all number-processing deficits are manifestations of generalized language disorders (e.g., Benson & Denckla, 1969; Benson & Weir, 1972; Berger, 1926; Collignon et al., 1977; Dahmen, Hartje, Bussing, & Sturm, 1982). Unfortunately, previous treatments of the general/specific issue have not been entirely adequate. In the first place, the nature of the posited general deficits has usually not been specified in sufficient detail. Before a general-deficit hypothesis can be evaluated, it must be set forth explicitly, in terms of damage to some component of a clearly specified normal system. The claim that a calculation deficit reflects a general spatial-processing impairment or that a number-comprehension deficit reflects a general language impairment is too vague to be meaningful. One must specify in detail the nature of the general mechanism presumed to be disrupted, how damage to the mechanism could produce the observed pattern of performance on number tasks, and what sorts of nonnumber deficits are expected given the hypothesized disruption.

Consider, for example, the hypothesis that patient R.R.'s deficit involves damage to a general mechanism that is responsible for retrieval of phonological forms in language production. Before this hypothesis can be considered seriously, it must be made more explicit. We have argued that in attempting to retrieve the phonological forms of number words, R.R. accesses the appropriate number lexical class (i.e., ones, teens, or tens) but often the wrong item within class. Thus, in R.R.'s case the general-deficit hypothesis would presumably state that, in retrieving the phonological form of any word, R.R. accesses the appropriate class but often the wrong item within class. What, though, are the lexical classes for nonnumber words, and what sorts of errors would we expect R.R. to make in the production of nonnumber words? Should each semantic category be considered a class, so that we should expect R.R., when trying to say "apple," to say the name of some other fruit? Or should lexical classes be identified with form classes, so that we should expect R. R. when trying to produce a verb, to sometimes produce another verb (but not necessarily one related in meaning to the intended verb)? Unless these and other questions are answered, we cannot begin to evaluate a general-impairment hypothesis.

As another example of the need to make hypotheses explicit, consider the concept of spatial dyscalculia. This concept reflects the widely held view that a general spatial-processing disorder is frequently the cause of a calculation impairment. However, the nature of the spatial-processing system that is presumed to be damaged, the ways in which the presumed disruption of the system impairs calculation, and the other deficits that should result from the spatial disorder have not been specified. Instead, a deficit has typically been labeled spatial dyscalculia whenever some aspect of the spatial arrangement of numbers in a calculation is incorrect.

In some instances, deficits labeled spatial dyscalculia do not seem consistent with any reasonable construal of the notion of a general spatial disorder. Consider the examples of patient 1373's performance shown in Fig. 6. In these examples the intermediate products are not aligned properly: 74 and 2156 should be shifted one column to the left. However, it is unlikely that the errors reflect a general spatial impairment that renders the patient unable to align numbers. The intermediate products are perfectly aligned—the alignment is simply incorrect. Further, the alignment of the digits in the sums of the intermediate products is perfect. This example points up the pitfalls involved in the use of vague notions like "spatial disorder" instead of a specific description of the nature of the presumed deficit.

We do not intend to imply that spatial disorders are never implicated in calculation impairments. Our point is simply that it is incumbent upon researchers who offer spatial deficit hypotheses to specify these hypotheses in sufficient detail that they may be evaluated.

A second point to be made about the general vs. specific deficit issue is that this issue is of concern only in certain circumstances. If one's aim is to specify exactly what component of the cognitive system is disrupted in a particular patient, then the general vs. specific issue is obviously relevant. However, if the aim is to use patterns of impaired performance to make inferences about the structure of the number-processing/calculation mechanisms, the general/specific issue is often irrelevant. For example, the issue is irrelevant for our use of R.R.'s performance to make inferences about the structure of the verbal number-production lexicon. Regardless of whether R.R.'s deficit is general or number specific, his performance implies an organization of the number-production lexicon into functionally distinct classes. This point is worth emphasizing, because it is often assumed that highly selective, pure deficits are needed before one can use the deficits to make inferences about the structure of the normal system. On this view, to study the number-processing/calculation system, one must have patients whose deficits are limited to number processing or calculation; patients with, say, general language impairments are not appropriate. This view, however, is mistaken. The pattern of performance within the number domain will often reveal aspects of the structure of the number-processing/calculation mechanisms even if the patient has a variety of nonnumber deficits. Patient R.R., for example, has severe deficits in language comprehension and production.

We suggest, then, that the question of whether a deficit is general or specific is less central than has often been supposed. In the first place one can, independent of this issue, consider what stage(s) of processing within the number-processing/calculation system are disrupted. For example, one can determine that a patient is impaired in retrieving arithmetic facts before taking up the question of whether the retrieval deficit is specific to arithmetic facts or more general. Further, if one's aim is not to characterize deficits but to elucidate the structure of number-system components, the general/specific issue may often be completely irrelevant. Thus, we suggest that the first step in a study involving number-processing/calculation deficits should be the identification of the disrupted stage(s) of processing (e.g., lexical processing in Arabic number comprehension; retrieval of arithmetic facts). Subsequently, if it is relevant, the general/specific issue may be considered. Initially, one may ask whether the pattern of performance within number tasks is consistent with a general deficit. In many instances it may be possible to reject a general-deficit hypothesis on the basis of number-task performance alone. Consider, for example, a patient who fails to carry properly when adding. The hypothesis of a general working memory disorder that renders the patient incapable of holding a carry digit in memory can be entertained only if the addition performance is consistent with the assumption that the patient often forgets carry digits, and the patient also shows deficits in other number-processing/calculation tasks that require temporary memory (e.g., carrying in multiplication, performing mental calculations in which operands and/or intermediate results must be maintained in memory). If the number-task performance is consistent with a general-deficit hypothesis, then appropriate nonnumber tasks can be employed to determine whether the deficit is indeed general. Of course, the general-deficit hypothesis must be sufficiently explicit that clear predictions can be generated at each step of the process.

CONCLUSION

We have focused in this article on the articulation of a framework for the analysis of number processing and calculation. Within this framework the primary objective is the development of a model that is sufficiently detailed to serve as a basis for explaining the number-processing/calculation performance of both normal and cognitively impaired subjects. A central assumption of our approach is that the performance of patients with acquired cognitive disorders has the same status as the performance of normal subjects in theory construction. To be sure, the specific methods for relating performance to assumptions about cognitive structures and processes are somewhat different for impaired performance than for unimpaired performance. Nevertheless, the two types of data have the same logical status in developing and evaluating models. A model of a cognitive system must not only be consistent with normal performance;

it must also be capable, when "lesioned" appropriately, of explaining patterns of impaired performance in patients with acquired cognitive disorders.

Working within this framework we have proposed a general model that posits autonomous cognitive systems for number processing and calculation. Within the number-processing system, we drew a distinction between number-comprehension and number-production mechanisms, and within each of these subsystems we further distinguished components for processing Arabic numbers from components for processing verbal numbers. We also proposed a distinction, within the Arabic and verbal comprehension and production components, between lexical- and syntactic-processing mechanisms. Finally, we distinguished between phonological- and graphemic-processing mechanisms within the lexical-processing components of the verbal system. Within the calculation system we distinguished among cognitive mechanisms for processing operation symbols or words, mechanisms for retrieval of basic arithmetic facts, and mechanisms for execution of calculation procedures.

The literature on dyscalculia, as well as results from our own studies, strongly supports our assumptions about the general architecture of the number-processing/calculation systems. Furthermore, the detailed analysis of a single case, R.R., provided grounds for inferences about the functioning of the verbal number-production system, and in particular about the organization of the phonological number-production lexicon. The analysis of this case demonstrates how detailed consideration of patients' performance can lead to progressively finer specification of the functioning of cognitive systems.

In this article we have focused exclusively on the cognitive analysis of number-processing and calculation deficits. However, we suggest that our approach is not only compatible with, but is in fact a necessary precondition for, work that uses cognitive deficits to infer brain/cognition relationships. The explication of the structure of the normal cognitive system and the ways in which damage to the system produces the observed patterns of impaired performance is an essential step in inferring relationships between cognitive processes and brain mechanisms. To establish meaningful cognitive deficit/neuropathology correlations (e.g., disruption of cognitive process x is associated with damage to neural mechanism y), one must understand in detail not only the relevant brain systems and the nature of the damage to these systems, but also the relevant cognitive processes and the nature of their disruptions.

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