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Cognitive mechanisms in numerical processing: Evidence from acquired dyscalculia*

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Abstract

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This article discusses cognitive neuropsychological research on acquired dyscalculia (i.e., impaired numerical processing resulting from brain damage), surveying issues of current interest, and illustrating the ways in which analyses of acquired deficits can contribute to an understanding of normal processing. I first review the logic whereby inferences concerning normal cognition are drawn from patterns of impaired performance. I then consider research exploring the general functional architecture of the cognitive numerical processing mechanisms, and finally turn to studies aimed at probing the internal structure and functioning of individual processing components.

When the brain is damaged through accident or disease, deficits in cognitive functioning frequently result. Traditionally, research on these "acquired" deficits has focused on defining clinical syndromes (constellations of co-occurring symptoms), and relating these syndromes to damage in particular brain areas. Recently, however, researchers have come to realize that systematic analyses of acquired cognitive deficits can also provide a basis for inferences about normal cognitive processes. This article discusses impairments in processing of numerical information (collectively referred to as *acquired dyscalculia*), exploring the insights offered by these impairments into the structure and functioning of normal numerical processing mechanisms.

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ASSUMPTIONS AND METHODS IN COGNITIVE NEUROPSYCHOLOGICAL RESEARCH

The logic whereby data from brain-damaged patients are brought to bear on issues concerning normal cognition is somewhat different from the logic underlying cognitive research with normal subjects. Hence it is worthwhile to consider the basic assumptions motivating cognitive neuropsychological research, and the methodological implications of these assumptions. For more detailed discussions, see Caramazza (1984, 1986), Caramazza and McCloskey (1988), McCloskey and Caramazza (1988), and Shallice (1979).

Fundamental assumptions

The basic premise of cognitive neuropsychological research is straightforward: the impaired cognitive performance of a brain-damaged patient reflects the functioning of a previously normal cognitive system with one or more components damaged. This premise incorporates two related assumptions. The first is that brain damage may have selective effects, disrupting some components of a cognitive system while leaving other components intact. The second assumption is that cognitive systems disrupted by brain damage do not undergo a functional reorganization in which a cognitive architecture substantially different from the normal architecture is created.

Given these assumptions, a pattern of impaired performance may be brought to bear on issues concerning normal cognition by asking: what must the normal cognitive system be like in order that damage to the system could result in just this performance pattern? In this way inferences may be drawn about the general functional architecture of a cognitive system (i.e., the major processing components and the interactions among them), and about the internal structure and functioning of individual components (i.e., the specific computations carried out, and the nature of the representations manipulated).

For example, Warrington (1982) drew conclusions about the general architecture of cognitive numerical processing mechanisms from the case of patient DRC, a physician who sustained left parietal–occipital damage when a blood vessel in his brain ruptured. DRC could read and write numbers without difficulty; he was able to judge rapidly and accurately which of two numbers was larger; and he could give reasonable estimates of numerical attributes (e.g., how tall is the average English woman?). Testing of basic arithmetic, however, revealed a deficit: in tasks requiring speeded responses DRC was much slower and somewhat less accurate than control subjects, even for very simple problems (e.g., 5 + 7). Questioned about his difficulties in solving simple addition and subtraction problems, DRC stated that he often knew the approximate answer to a problem, but no longer knew the exact answer. DRC claimed, however, that he understood arithmetic operations, and that he could work out answers to problems by counting (e.g., solving $8+4$ by counting up 4 from 8). Consistent with these claims, DRC was able to give sensible definitions of addition, subtraction, multiplication, and division, and performed well in untimed tests of arithmetic problem-solving.

Warrington (1982) interpreted these results as evidence for a distinction between number knowledge and arithmetic knowledge. Given this distinction, she argued, DRC's performance could be explained at a general level by assuming that his arithmetic knowledge was disrupted while his number knowledge remained intact. Warrington (1982) further suggested that DRC's performance motivates a distinction within arithmetic knowledge between knowledge of arithmetic operations, and knowledge of arithmetic facts. DRC's knowledge of arithmetic operations was apparently intact, as evidenced by his ability to define the operations, and to solve problems in untimed testing. In contrast, DRC was impaired in retrieving stored knowledge of arithmetic "table" facts such as $8+4=12$. As a consequence, the facts had to be worked out (e.g., by counting), leading to impairment on speeded tasks (although allowing for good performance on untimed tasks).

Single-patient studies

As Warrington's (1982) study illustrates, the appropriate methodology in cognitive neuropsychological research is that of the single-patient study (Caramazza, 1986; Caramazza & McCloskey, 1988; McCloskey & Caramazza, 1988). In studies with normal subjects, data are typically averaged over groups of subjects, in order to minimize the noise (i.e., measurement error) in the data brought to bear on theoretical issues. Averaging over subjects requires the assumption that the subject group is homogeneous with respect to the cognitive mechanisms under investigation. Given this assumption, any differences among subjects in patterns of performance may be considered noise, and averaging serves to produce "cleaner" measures than those from individual subjects. Thus, averaging over subjects is simply an expedient method for reducing error of measurement.

This methodological expedient is not available in studies of brain-damaged patients, because it cannot be assumed that groups of patients are homogeneous with respect to the cognitive mechanisms of interest. One may assume that *premorbidly* (i.e., prior to brain damage) the patients in a group were homogeneous, in any circumstances where an assumption of homogeneity would be justified for normal subjects. However, one cannot assume that damage to the cognitive mechanisms under investigation is uniform across patients; brain damage may disrupt a cognitive system in a variety of different ways. Therefore, differences in performance patterns among brain-damaged patients cannot be dismissed as noise; these differences may reflect, at least in part, differences in forms of impairment.

Accordingly, averaging data over brain-damaged patients is inappropriate, unless the patients show the same pattern of performance in all tasks presented in a study. However, when testing and data analyses are carried out at a level of detail adequate to address issues of current interest in cognitive science, two or more patients will rarely show the same performance pattern across all tasks. In most instances, therefore, single-patient studies provide the only valid basis for inferences about normal cognition.

Generalizing from single-patient studies

Drawing conclusions from a single-patient study is, at least implicitly, a two-step process. First, one interprets the obtained pattern of performance by jointly postulating some characteristics of the patient's previously normal cognitive mechanisms, and functional damage to these mechanisms that would lead to the observed pattern of performance. Then, on the assumption that the cognitive mechanisms under investigation are shared by some population of normal individuals, the inferences about the patient's previously normal mechanisms are generalized to that population. For example, Warrington's (1982) argument from DRC's performance that knowledge of arithmetic facts should be distinguished from knowledge of arithmetic operations was presumably intended to apply not solely to DRC, but more broadly to the population of normal educated adults.

A concern often expressed regarding generalization from single-patient studies is that normal cognitive mechanisms may vary across individuals, so that inferences about a patient's premorbid cognitive system may not in fact generalize to people at large. Although the possibility of individual differences must indeed be borne in mind, the problems raised by this possibility are neither particularly serious, nor unique to single-subject studies.

The use of a single-case methodology does not mean that a theory of normal processing is developed on the basis of data from a single patient, any more than a theory is developed on the basis of data from a single experiment with normal subjects. Although empirical reports often present findings from only one or two patients, data from multiple single-patient studies are ultimately brought to bear in formulating and evaluating theories. If, then, there is substantial individual variation in the normal cognitive mechanisms under investigation, this variation should be signalled by a failure of results from various patients to converge in the conclusions they suggest about normal processing mechanisms.

For the most part, potential individual differences may be treated in the same manner in cognitive neuropsychological research as in research involving groups of normal subjects. When individual differences seem unlikely *a priori*, and available data provide no indication of such differences, conclusions may reasonably be generalized to the population at large. In circumstances where individual differences appear more likely, however, caution should be exercised in generalizing from a single-patient study (as well as in averaging over normal subjects).[1]

In general a single-patient study may be considered roughly comparable – in evidential weight and in limitations – to a single experiment with a group of normal subjects. That is, a single-patient study provides a valid basis for inferences about normal cognitive mechanisms; however, just as converging results from multiple experiments with normal subjects provide stronger evidence than results from a single experiment, converging findings from multiple single-patient studies are stronger than data from a single study.

COGNITIVE NEUROPSYCHOLOGICAL RESEARCH ON ACQUIRED DYSCALCULIA

The literature on acquired dyscalculia is extensive, dating back at least to the early years of this century (Lewandowsky & Stadelmann, 1908; for recent reviews, see Boller & Grafman, 1983; Levin & Spiers, 1985; Spiers, 1987). However, as in other areas of neuropsychological research, many of the available studies were either not directed at elucidating the structure of normal processing mechanisms, or were not conducted in light of current cognitive theory. Accordingly, much of the literature on dyscalculia is of limited value for purposes of drawing inferences about normal numerical processing. For example, a substantial proportion of the studies report only results averaged, or otherwise aggregated, over patients. Further, in some of the studies presenting findings from individual

[1]One type of individual variation does require special consideration in single-subject studies. Suppose that although nearly all individuals in a population are essentially the same with respect to the cognitive mechanisms under investigation, a few individuals are qualitatively different. In a group study of normal subjects, the inclusion of a small proportion of atypical subjects will probably have little effect on group averages, and the averages will accurately reflect the structure and functioning of the cognitive mechanisms shared by the vast majority of the population. On the other hand, if a single-subject study is undertaken with one of the highly atypical individuals, then incorrect conclusions about the population may be drawn. In cognitive neuropsychological research the specific concern is that a patient's cognitive system may have been atypical prior to brain damage. (The possibility that a patient may have an unusual form of damage is not a concern, because one seeks to generalize not conclusions about forms of damage, but rather conclusions about patients' premorbid cognitive systems.) Although specific to single-subject studies, problems associated with highly atypical cognitive systems are probably not especially serious. These problems may be minimized by taking care in selecting subjects (e.g., by avoiding patients for whom there is evidence of developmental cognitive deficits). Further, given the assumption that atypical cognitive systems are present in only a small proportion of the normal population, it seems likely that only a small proportion of single-case studies would involve patients with atypical premorbid systems. The results of these studies would presumably diverge in their implications from the bulk of the available studies, and, like anomalous results in any area of scientific research, would presumably have little influence on theory development.

patients, the discussions are brief and anecdotal, so that crucial details about procedures and results cannot be ascertained.

Fortunately, recent years have seen a growing interest in dyscalculia among cognitive neuropsychologists seeking to develop and evaluate models of normal processing, and a substantial number of carefully conducted and systematically reported single-patient studies have recently appeared in the literature (e.g., Cohen & Dehaene, 1991; Ferro & Botelho, 1980; McCloskey, Sokol, & Goodman, 1986; Warrington, 1982). These studies, as well as a few strong efforts from earlier years (e.g., Singer & Low, 1933), provide a basis for exploring several basic issues concerning normal numerical processing.

In this article I attempt to highlight issues of current interest in cognitive neuropsychological research on acquired dyscalculia, and to illustrate some of the ways in which analyses of acquired deficits can contribute to an understanding of normal numerical processing. I first consider research exploring the general functional architecture of the cognitive numerical processing mechanisms, and then turn to studies aimed at probing the internal structure and functioning of individual processing components.

FUNCTIONAL ARCHITECTURE OF COGNITIVE NUMERICAL PROCESSING MECHANISMS

What is the nature of the mental machinery underlying basic numerical abilities? A simple model proposed by McCloskey, Caramazza, and Basili (1985) provides a useful starting point in addressing this question.

A model of numeral processing and calculation

The McCloskey et al. (1985) model considers at a general level the cognitive mechanisms mediating comprehension and production of arabic and verbal numerals,[2] and execution of simple calculations.

Numeral-processing mechanisms

As illustrated in Figure 1, the model posits functionally independent *numeral comprehension* and *numeral production* mechanisms. Numeral comprehension

[2]I use the term *numeral* to refer to a symbol or set of symbols representing a number. *Arabic numerals* are numerals in digit form (e.g., 56), and *verbal numerals* are numerals in the form of words (e.g., *fifty-six*), whether the words are spoken or written. Finally, I denote verbal numerals in spoken and written form as *spoken verbal numerals* and *written verbal numerals*, respectively. In using the term *numerals* I depart from the terminology of McCloskey et al. (1985), who refer to arabic and verbal numbers. However, the present nomenclature is helpful in maintaining the distinction between numbers, and symbols representing numbers.

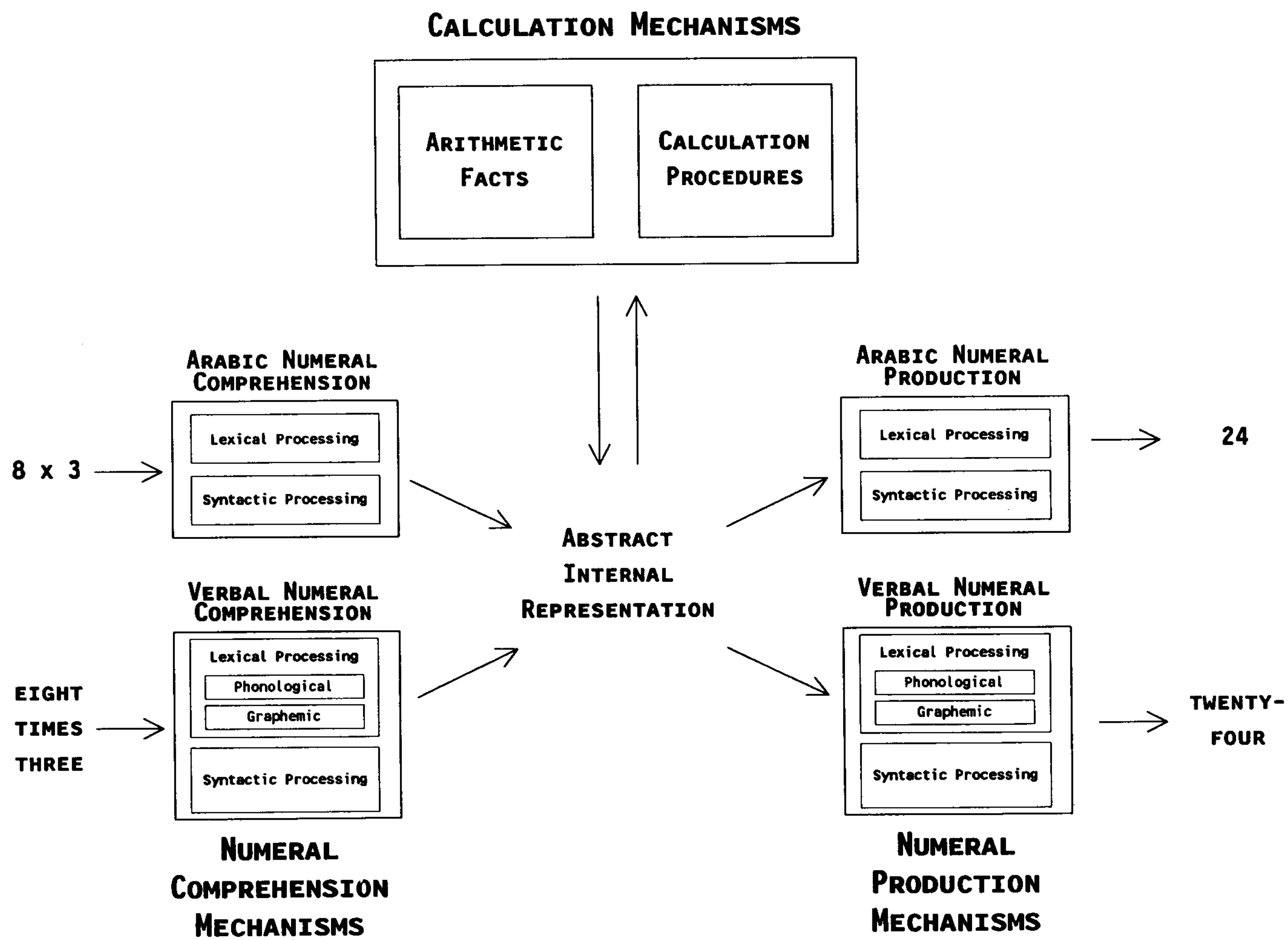


Figure 1. *Schematic depiction of the major processing components posited by the McCloskey, Caramazza, and Basili (1985) model of numeral processing and calculation.*

mechanisms convert numerical inputs into internal semantic representations for use in subsequent cognitive processing, such as performing calculations. Numeral production mechanisms translate internal representations of numbers into the appropriate form for output.

The internal semantic representations are assumed to specify in abstract form the basic quantities in a number, and the power of ten associated with each. For example, the arabic numeral comprehension process is assumed to generate from the stimulus 5030 the semantic representation {5}10EXP3, {3}10EXP1. The digits in braces (e.g., {5}) indicate quantity representations and 10EXP*n* indicates a power of 10 (e.g., 10EXP3 specifies 10 to the third power, or thousand). Thus, {5}10EXP3, {3}10EXP1 indicates a number made up of five thousands and three tens. This particular notation is adopted merely to avoid confusion between internal semantic representations of numbers, and arabic or verbal numerals. The important assumption is that the internal representations specify basic quantities and their associated powers of 10. (See Dehaene, Dupoux, & Mehler, 1990, for a contrasting view of internal numerical representation.)

In addition to distinguishing numeral comprehension and production mechanisms, the McCloskey et al. (1985) model further divides these mechanisms into components for processing *arabic numerals* (i.e., numerals in digit form, such as 362), and components for processing *verbal numerals* (i.e., numerals in word form, such as *three hundred sixty-two*). For example, reading a price tag would implicate arabic numeral comprehension processes, whereas writing a check would involve both arabic and verbal numeral production processes.[3]

Within the arabic and verbal numeral comprehension and production components, a further distinction is drawn between *lexical* and *syntactic* processing mechanisms. Lexical processing involves comprehension or production of the individual elements in a numeral (e.g., the digit *3* or the word *three*), whereas syntactic processing involves processing of relations among elements (e.g., word order) in order to comprehend or produce a numeral as a whole. For example, translation of the verbal numeral *six hundred forty* into the semantic representation {6}10EXP2, {4}10EXP1 would require lexical processing to generate internal representations for the words *six*, *hundred*, and *forty* (e.g., {6}10EXP0, 10EXP2, and {4}10EXP1, respectively), as well as syntactic processing to determine that because *hundred* followed *six* in the stimulus numeral, the quantity {6} should be associated with the power-of-ten marker 10EXP2 in the final semantic representation.

[3]In referring to verbal numeral comprehension and production components, my colleagues and I did not intend to claim that the mechanisms for processing numbers in word form are necessarily separate from the mechanisms for processing language in general; this remains an open question. The approach we adopted was to describe cognitive mechanisms in terms of the role these mechanisms play in number processing and calculation, leaving open the question of whether some of the mechanisms are also involved in other cognitive processing. See pp. 152–153 for further discussion.

Finally, within the lexical processing components for verbal numeral comprehension and production, the McCloskey et al. (1985) model distinguishes between *phonological* processing mechanisms for processing spoken number words and *graphemic* processing mechanisms for processing written number words. For example, spoken production of the word *six* would require retrieval of a phonological representation (i.e., /sIks/) from a phonological output lexicon, whereas written production of this word would require retrieval of a graphemic representation (i.e., S-I-X) from a graphemic output lexicon. No phonological/graphemic distinction is drawn for syntactic processing; the same syntactic processing mechanisms are assumed to underlie processing of both spoken and written verbal numerals.

Calculation mechanisms

Performing calculations requires, in addition to numeral comprehension and production, cognitive processes specific to arithmetic. In particular, the McCloskey et al. (1985) model posits components for comprehension of operation symbols (e.g., +) and words (e.g., *plus*), retrieval of arithmetic "table" facts, and execution of calculation procedures.

Consider, for example, the following problem:

$$\begin{array}{r} 64 \\ \underline{\times 59} \end{array}$$

According to the McCloskey et al. (1985) model, processing of the operation sign (×) would lead to activation of the multiplication procedure. This procedure, which provides an ordered plan for the solution of multiplication problems, would call first for processing of the digits in the rightmost column (i.e., 4 and 9). Thus, arabic numeral comprehension processes would be recruited to translate the digits into abstract internal representations. These representations, along with a representation of the arithmetic operation, would then be taken as input by the arithmetic fact retrieval process, which would return an abstract internal representation of the product (i.e., {3}10EXP1, {6}10EXP0). The multiplication procedure would then call for the ones portion of the product to be written in arabic form beneath the rightmost column of the problem. Thus, arabic numeral production mechanisms would translate the abstract representation {6}10EXP0 into a representation of the digit 6, and general writing processes would produce the written output from this digit representation. Processing would continue in this fashion until all partial products had been computed. At this point the addition procedure would be called, and the partial products would be summed and written under control of this procedure.

Empirical evidence

Studies demonstrating that some aspects of numeral-processing and calculation may be disrupted selectively provide support for many of the distinctions among processing components drawn by McCloskey et al. (1985), as the examples discussed in the following sections illustrate.

Patterns of impaired numeral processing

Selective impairment of numeral production. Benson and Denckla (1969) reported the case of a 58-year-old man with left hemisphere damage apparently resulting from a cerebrovascular accident, or CVA (i.e., disruption of blood flow through a cerebral artery, usually due to blockage or rupture of the vessel). When simple arithmetic problems (e.g., $4 + 5$) were presented visually in arabic form or aurally in verbal form the patient consistently chose the correct answer from a multiple-choice list of arabic numerals. This result implies an ability to comprehend the arabic numerals in the written stimulus problems and in the multiple-choice lists, as well as an ability to comprehend the spoken verbal numerals in the aurally presented problems. Further, the patient was able to point to the correct arabic numeral when a verbal numeral was dictated, again suggesting intact comprehension of arabic and spoken verbal numerals.

On tasks requiring production of arabic or spoken verbal numerals, however, the patient was severely impaired. When asked to say or write the answer to simple arithmetic problems, he was often incorrect. For the problem $4 + 5$, for example, he said "eight" and wrote "5" (but chose "9" from a multiple-choice list). The patient's excellent performance on multiple-choice arithmetic problems suggests that the errors on problems requiring written or spoken answers reflect an impairment in producing the responses, and not in comprehending the stimuli or in retrieving the relevant arithmetic table facts. Consistent with this conclusion, the patient made frequent errors when asked to write arabic numerals to dictation (e.g., stimulus "two hundred twenty-one," response 215), and when asked to read aloud arabic numerals. Given the above-described evidence of intact numeral comprehension, these numeral reading and writing errors are unlikely to be errors in comprehending spoken verbal stimuli (writing-to-dictation task) or arabic stimuli (reading task); rather, the errors presumably stem from deficits in production of Arabic and spoken verbal numerals. Thus, Benson and Denckla's (1969) patient shows a dissociation between comprehension (intact) and production (impaired) for both arabic and verbal numerals, supporting the assumption that numeral comprehension and numeral production mechanisms are functionally distinct.

Selective impairment of syntactic processing in arabic numeral production. A more specific dissociation is apparent in results reported by Singer and Low

(1933). These researchers studied a 44-year-old man who suffered brain damage as a result of carbon monoxide poisoning. The patient's performance in production of arabic numerals revealed a dissociation in which lexical processing was intact but syntactic processing was impaired. In writing arabic numerals to dictation the patient was uniformly correct for 1- and 2-digit numerals. For larger numerals, the individual non-zero digits were consistently correct, but the responses were of the wrong order of magnitude. For example, "two hundred forty-two" was written as 20042, and "two thousand five hundred" as 2000500.

These errors apparently did not result from difficulty in comprehending the dictated stimuli, because the patient was invariably correct in judging which of two spoken verbal numerals was larger, and in selecting from a multiple-choice list the arabic numeral matching a spoken verbal numeral. Hence, the writing-to-dictation errors apparently reflected a deficit in arabic numeral production, and more specifically a syntactic processing deficit. The excellent performance for 1- and 2-digit numerals, and the consistent production of the correct non-zero digits in larger numerals, suggest that lexical processing mechanisms for arabic numeral production were intact: for each non-zero quantity in a number, the patient was able to retrieve the appropriate digit representation. However, syntactic processing was disrupted: the patient was impaired in arranging the non-zero digits into the proper positions and combining them with appropriately positioned placeholder 0's to produce the correct arabic representation of the entire number.

Patterns of impaired calculation

Selective impairment of operation symbol comprehension. Ferro and Botelho (1980) described two Portuguese patients (AL and MA) with selective deficits in comprehension of written operation symbols (e.g., +, ×). When single- and multi-digit arithmetic problems were presented in written form both patients often performed (correctly) the wrong operation. For example, AL multiplied when presented with 721 + 36, obtaining the correct product 25,956. The fact that incorrect operations were performed correctly suggests that the patients were intact in comprehension and production of arabic numerals, in retrieval of arithmetic facts, and in execution of calculation procedures. Thus, the performance pattern suggests a deficit in comprehension of operation symbols, and indeed several tests of operation symbol comprehension revealed clear deficits in both patients. Interestingly, the operation comprehension deficit was specific to the written operation symbols: When arithmetic problems were presented aurally, the patients performed well, and in particular had no difficulty comprehending the spoken operation words "plus," "times," and so forth.

Selective impairment of arithmetic fact retrieval. I have already discussed Warrington's (1982) study of DRC, which suggests that arithmetic fact retrieval may be selectively disrupted. A study speaking more specifically to the distinction

Figure 2. *Examples of PS's fact retrieval errors on multi-digit multiplication problems. (A) In this problem PS retrieved 32 as the answer to 9×4, and 72 as the answer to 9×9. (B) In this problem PS retrieved 54 as the answer to 7×9.*

between arithmetic fact retrieval and execution of calculation procedures has been reported by Sokol, McCloskey, Cohen, and Aliminosa (1991). Sokol et al. studied patient PS, a 40-year-old college-educated financial planner who had suffered a left hemisphere CVA. In single-digit multiplication PS was clearly impaired, erring on 451 of 2300 problems (20%). Her excellent performance on several numeral-processing tasks indicated that the multiplication errors could not be attributed to numeral comprehension or production deficits, and reflected instead a deficit in retrieving multiplication table facts. This fact retrieval deficit was also apparent when PS was asked to solve multi-digit multiplication problems, as the examples in Figure 2 illustrate.

In execution of calculation procedures, however, PS was clearly intact. When solving the multi-digit problems she consistently carried out the appropriate single-digit fact retrievals (although not always successfully) in the appropriate order. Further, she dealt appropriately with partial products (e.g., writing the ones digit and carrying the tens digit, aligning the rows correctly), and executed the addition procedure appropriately when adding these products (although she occasionally erred in retrieval of addition facts). Thus, PS presented with a clear dissociation between retrieval of arithmetic facts (impaired) and execution of calculation procedures (intact).

Additional mechanisms and alternative architectures

The McCloskey et al. (1985) model reflects the view that in developing cognitive theories it is advantageous to begin with a simple, tightly constrained model. Such a model generates straightforward predictions, and provides a well-defined anchor point from which further theory development may proceed. Thus, the McCloskey et al. (1985) model postulates what might be considered a minimal repertoire of cognitive mechanisms for accomplishing Arabic and verbal numeral processing, and basic arithmetic. Further, the flow of information is tightly constrained – the various processing components are assumed to communicate via a single form of internal numerical representation.

This simple model raises several sets of issues concerning numerical processing. For example, one might ask how or whether the postulated cognitive architecture could be extended to encompass numerical processing not considered in the current model (e.g., comprehension and production of roman numerals). However, the issues I will focus on here concern the adequacy of the model's assumptions about the processes currently within its purview. Is the postulated cognitive architecture adequate to account for processing of arabic and verbal numerals, and basic arithmetic? Or is there instead a need to posit additional processing mechanisms, or perhaps even to adopt an entirely different theoretical framework? In the following discussion I first consider a proposal suggesting that additional mechanisms are needed, and then examine a view of numerical cognition radically different from the view reflected in the McCloskey et al. (1985) model.

Asemantic transcoding alogorithms

The McCloskey et al. (1985) model holds that numerical transcoding (i.e., translating from one numeral form to another, such as from arabic to verbal numerals) is accomplished by a comprehension process that converts the input form into an internal semantic representation, followed by a production process that converts the semantic representation into the output numeral form. For example, reading aloud an arabic numeral (e.g., 3020) is assumed to involve an arabic numeral comprehension process that converts the arabic numeral into an internal semantic representation (e.g., {3}10EXP3, {2}10EXP1), and then a verbal numeral production process that converts the semantic representation into a sequence of (phonological) number word representations (e.g., *three thousand twenty*).

Deloche and Seron (1987) have suggested, however, that transcoding is accomplished by "asemantic" transcoding algorithms that translate from one numeral form to another without computing a semantic representation. In particular, they describe algorithms for arabic-to-verbal and verbal-to-arabic transcoding, and interpret transcoding errors in brain-damaged patients in terms of disruption to these algorithms (Deloche & Seron, 1982a, 1982b, 1984, 1987; Seron & Deloche, 1983, 1984).

In evaluating the Deloche and Seron (1987) proposal it is important first to recognize that the question is not whether asemantic transcoding algorithms should be posited *instead* of the numeral comprehension and production processes assumed by McCloskey et al. (1985), but whether the transcoding algorithms should be postulated *in addition to* these numeral comprehension and production processes. Although Deloche and Seron (1987) focus exclusively on transcoding tasks in discussing numeral processing, transcoding is not the sole, or even the primary, computational task that numeral processing mechanisms must accomplish. In many if not most instances where people process numerals, the meaning of the numerals is centrally implicated. For example, when someone examines the arabic numeral on a price tag, the aim is not usually to read the numeral aloud, but rather to determine its meaning, in order to answer such questions as: is this a reasonable price for this product? Answering questions of this sort presumably requires a process that converts the arabic numeral on the price tag into an internal semantic representation. Similarly, in situations where numerals are spoken or written the process often begins with a to-be-expressed meaning, and not with an external numeral stimulus that is to be translated to a different form. Thus, regardless of how numeral transcoding operations are carried out, one must still postulate comprehension processes that convert numeral inputs into semantic representations, and production processes that convert semantic representations into arabic or verbal numerals.

Therefore, the issue is whether there is reason to postulate algorithms dedicated specifically to translation of numerals from one form to another, when numeral comprehension and production processes sufficient to accomplish the translations must in any case be postulated for other reasons. In considering this issue it is important to note that arabic-to-verbal and verbal-to-arabic transcoding are not simple processes involving one-to-one translation of digits to words, or vice versa. The mapping between arabic and verbal numerals is quite complex. For example, in converting arabic to verbal numerals the same digit (e.g., 2) may map onto different number words (e.g., *two*, *twelve*, *twenty*) depending on where in the arabic numeral it appears. In some instances two digits (e.g., the 1 and 2 in 12 or 12,000, but not 120) correspond to a single word (e.g., *twelve*). Further, 0's in an arabic numeral have no verbal realization, unless they appear in isolation or to the right of a 1 in certain positions in a number (in which case the 1 and 0 are realized together as *ten*, as in 210 or 210,000, but not 2100). Finally, the transformation of arabic to verbal numerals requires the insertion of multiplier words (e.g., *hundred*, *thousand*) at appropriate points in the word sequence, although these words do not correspond to particular digits in the arabic numeral.

Thus, it is not the case that simple and direct transcoding algorithms can be defined to shortcut a more laborious process of converting from an input numeral form to a semantic representation, and then from a semantic representation to an output numeral form. Rather, as Deloche and Seron's (1987) discussion illustrates, the transcoding algorithms must be comparable in complexity to the numeral comprehension and production processes postulated by the McCloskey et al. (1985) model. Indeed, the arabic-to-verbal transcoding algorithm proposed by Deloche and Seron (1987; see also Cohen & Dehaene, 1991) is similar in many respects to the verbal numeral production algorithm sketched by McCloskey et al. (1985) and developed in more detail in McCloskey et al. (1986).

Turning next to empirical motivation, Deloche and Seron (1987) offer no specific evidence that transcoding is accomplished by asemantic algorithms as opposed to numeral comprehension and production processes. They discuss various types of errors made by French brain-damaged patients in transcoding arabic numerals to French verbal numerals, and vice versa, suggesting that these errors can be interpreted in terms of disruption of the posited transcoding algorithms. However, the errors could also be interpreted by reference to disruption of numeral comprehension and production processes. For example, several of the error types attributed to disruption of the arabic-to-verbal transcoding algorithm are interpreted by McCloskey et al. (1986) in terms of damage to a verbal numeral production process. Cohen and Dehaene (1991) present results from a patient who was impaired in arabic-to-spoken-verbal transcoding, and interpret the patient's performance in terms of an asemantic transcoding process. However, as discussed in a later section (pp. 142–145), the evidence they present is less than definitive.

The hypothesis of asemantic transcoding clearly merits further investigation. In exploring this hypothesis it might be worthwhile to consider not only the possibility of complete asemantic transcoding algorithms, but also more limited forms of asemantic transcoding. At the simplest level one might consider whether representations of individual digits (e.g., 6), and phonological and orthographic representations of individual number words (e.g., /sIks/, S-I-X) may be mapped onto one another without an intervening semantic representation. At the least, translation between phonological and graphemic number-word representations may presumably be accomplished through the grapheme–phoneme and phoneme–grapheme conversion processes postulated for words in general by most models of reading (e.g., Patterson & Morton, 1985) and spelling (e.g., Ellis, 1982), although these processes would reliably generate correct translations only for number words with regular spelling–sound correspondences, such as *seven*, and not for irregular words such as *one*. Mapping between phonological and orthographic number-word representations might also be accomplished via the direct phonology-to-orthography and orthography-to-phonology routes incorporated in some reading and spelling models (e.g., Bub, Cancelliere, & Kertesz, 1985; Goodman & Caramazza, 1986). Conceivably, analogous routes might carry out mappings between digit representations and phonological or graphemic number-word representations.[4]

[4]Campbell and Clark (1988) discuss results from a patient tested by my colleagues and myself that seem to – but in fact do not – provide evidence of asemantic mapping between number words and digits. McCloskey et al. (1985) reported that patient HY was at chance in judging which of two written number words was larger (e.g., *seven* vs. *five*), suggesting impaired comprehension of the words. McCloskey et al. (1986), however, reported that HY performed well in matching individual written number words with their arabic equivalents (e.g., *seven* and 7), suggesting an intact ability to translate between the two numerical codes. These two results seem to suggest that HY could map number words onto digits while at the same time being unable to understand (i.e., derive a semantic representation for) the words. However, this was not in fact the case. The magnitude comparison

At a slightly higher level, it might be hypothesized that people learn some basic asemantic transcoding rules for mapping between "simple" multi-digit arabic numerals – that is, numerals comprising a non-0 digit followed by one or more 0's – and their verbal counterparts (e.g., $x000 \leftrightarrow x$ thousand, as in $6000 \leftrightarrow$ six thousand). Rules of this sort would suffice for arabic-verbal transcoding of simple numerals, but would not be adequate for transcoding more complex numerals (e.g., $4012 \leftrightarrow$ four thousand twelve), and hence would not constitute complete asemantic transcoding algorithms. Although there is currently no clear empirical support for asemantic transcoding of simple larger numerals, the verbal-to-arabic transcoding errors made by Singer and Low's (1933) patient (see pp. 116–117) might be interpreted in terms of such rules: Unable to carry out normal arabic numeral production, the patient may have relied upon direct mapping rules such as x thousand $\leftrightarrow x000$ and x hundred $\leftrightarrow x00$, leading to errors such as 2000500 written in response to "two thousand five hundred." (See also the discussion of term-by-term transcoding errors in Deloche & Seron, 1987).

Clarification of the roles of semantic and asemantic processes in numerical cognition will require not only additional data, but also (and perhaps more importantly) more detailed and explicit formulations of the various theoretical positions. With respect to asemantic transcoding, effort might profitably be focused on articulating further the transcoding algorithms described by Deloche and Seron (1987; see also Cohen & Dehaene, 1991), and perhaps also the more limited processes sketched above. At the same time, there is a clear need to elaborate the McCloskey et al. (1985) hypothesis of numeral comprehension and production processes that communicate via semantic representations of numbers, by formulating more explicit assumptions concerning how the various processes are carried out. As discussed in a later section, some progress in this direction has been made with regard to verbal numeral production, but the internal structure and functioning of the other numeral processing components remain to be specified.

The encoding complex view: An alternative framework

A view of numerical processing very different from that of McCloskey et al. (1985) has been put forth by Campbell and Clark (1988; Clark & Campbell,

results showing impaired performance were obtained in preliminary testing of HY in September 1983. The study reported in McCloskey et al. (1986), however, was carried out several months to a year later, from April to September 1984. As is often the case in testing of brain-damaged patients, HY's performance changed over time; in particular, his processing of verbal numerals improved considerably. When tested on verbal numeral magnitude comparison during the period in which he performed well in arabic–verbal matching, HY was 95% correct (40/42). Thus, the McCloskey et al. (1986) results do not show a dissociation in which HY was unable to judge which of two number words was larger, yet was able to match a number word with its arabic equivalent.

1991). The McCloskey et al. model postulates a modular functional architecture in which autonomous processing components communicate via a single form of internal numerical representation. In contrast, Campbell and Clark's *encoding complex* view posits a non-modular architecture in which multiple numerical codes activate one another in the course of numeral processing and arithmetic tasks. The codes, which may include phonological, graphemic, visual, semantic, lexical, articulatory, imaginal, and analogue representations (Campbell & Clark, 1988, p. 209), are assumed to be interconnected in an associative network, so that individual codes may activate one another to produce a multi-component "encoding complex."

In the McCloskey et al. (1985) model, particular representational formats are constrained to be involved only at particular stages of particular numeral processing and calculation tasks. For example, phonological number-word representations are assumed to be implicated only in the numeral comprehension stage of tasks involving stimuli in the form of spoken verbal numerals, and the numeral production stage of tasks involving spoken verbal responses. The encoding complex view, in contrast, does not embody such constraints. Campbell and Clark apparently assume that any code may potentially be recruited at any phase of a numeral processing or calculation task, and that multiple codes may be implicated at any point of the processing. Furthermore, Campbell and Clark (1988; Clark & Campbell, 1991) posit individual differences in the complex of codes involved in particular tasks.

Although intriguing in many respects, Campbell and Clark's encoding complex view has not yet been developed into a specific model capable of generating clear predictions. As Sokol, Goodman-Schulman, and McCloskey (1989) pointed out in a recent critique, Campbell and Clark have not fully delineated the set of codes presumed to be implicated in numeral processing or calculation; they have not specified what codes are used in what tasks; they have not specified how the codes are used to accomplish the tasks; and they have not specified the nature of the presumed individual differences.

Empirical evidence

Although the encoding complex view does not represent a well-articulated alternative to the McCloskey et al. (1985) model, it is nevertheless important to examine the empirical evidence Campbell and Clark offer as grounds for adopting the encoding complex position and rejecting the sort of functional architecture postulated by McCloskey et al. For the most part this evidence concerns the forms of numerical representations implicated in particular numeral processing or calculation tasks.

The McCloskey et al. (1985) model assumes that central processing of numbers is carried out on a single form of abstract semantic representation. However, Campbell and Clark, as well as several other researchers, have argued that the available evidence points to the conclusion that numerical processing involves multiple format-specific codes – that is, codes tied to the forms in which stimuli are presented and/or the forms in which responses are elicited (e.g., Campbell & Clark, 1988; Clark & Campbell, 1991; Deloche & Seron, 1987; Gonzalez & Kolers, 1982, 1987; Kashiwagi, Kashiwagi, & Hasegawa, 1987; Tzeng & Wang, 1983; Vaid, 1985; Vaid & Corina, 1989).

It is certainly the case, as Clark and Campbell (1991) point out, that people have the capacity to manipulate format-specific numerical representations. For example, one can think of rhymes for number words (e.g, *seven* rhymes with *heaven*), or make images of digits. Further, some individuals report elaborate visual–spatial representations for number sequences (e.g., visualizing the number line from 0 to 100 as a spiral; see Seron et al., this issue). The question to be considered here, however, is whether format-specific representations play a functional role in arithmetic or other numerical processes that do not specifically require such representations. Although a detailed discussion is beyond the scope of the present article, a brief survey of the major findings cited in support of this view suggests that the seemingly impressive evidence is something less than definitive. (For further discussion see Sokol, Goodman-Schulman, & McCloskey, 1989; Sokol et al., 1991.)

Arithmetic with arabic and roman numerals. Gonzalez and Kolers (1982, 1987) measured RT for true/false responses to simple addition problems presented with correct or incorrect answers (e.g., 3 + 4 = 7, 2 + 6 = 9). Stimuli were presented in the form of arabic numerals (e.g., 3 + 4 = 7), roman numerals (e.g., III + IV = VII), or various arabic/roman combinations (e.g., 3 + IV = VII, III + 4 = 7). Performance varied as a function of problem format, leading Gonzalez and Kolers to conclude that subjects performed the task not by translating both arabic and roman numerals into abstract representations, but rather by operating upon representations tied to the physical form in which the numbers were presented.

Sokol et al. (1991) questioned the Gonzalez and Kolers (1982, 1987) conclusion, pointing out that these researchers failed to articulate explicit claims about how representations tied to roman or arabic stimulus formats were used in the arithmetic task, or how the use of these format-specific representations led to the obtained pattern of results. Presumably Gonzalez and Kolers did not intend to suggest that addition facts are stored separately in arabic form (e.g, 5 + 4 = 9) and in roman form (e.g., V + IV = IX), so that different stored facts were retrieved for roman and arabic problems. Yet if this was not the intended argument it is difficult to imagine what was in fact intended.

Sokol et al. (1991) also suggested that most if not all of the Gonzalez and Kolers findings could be interpreted simply by assuming that roman numerals take longer to comprehend (i.e., translate into abstract internal representations) than arabic digits. For example, this assumption provides a straightforward explanation for the finding that RT increased with the number of roman numerals in a stimulus item (e.g., RT was faster for $4 + 5 = 9$ than for $4 + V = IX$, which in turn was faster than $IV + V = IX$). Although Gonzalez and Kolers (1982, 1987) presented results they took as evidence against this slow-roman-comprehension interpretation, Sokol et al. (1991) argued that this evidence was weak at best.

Bilingual arithmetic. In support of the view that format-specific representations are implicated in arithmetic, Gonzalez and Kolers (1982, 1987; see also Clark and Campbell, 1991) also cited studies of bilinguals' arithmetic performance (Marsh & Maki, 1976; McClain & Huang, 1982), concluding that arithmetic implicates language-specific representations, and not "some metalinguistic mentalese" (Gonzalez & Kolers, 1987, p. 36). However, the bilingual arithmetic studies do not in fact warrant this conclusion. In the Marsh and Maki (1976) study, addition problems were presented in arabic form to bilingual subjects, and the subjects gave spoken responses in their preferred or non-preferred language. The number of addition operations required by a problem varied from one (e.g., $2 + 3$) to three (e.g., $4 + 3 + 2 + 5$). The McClain and Huang (1982) study was very similar, except that problems were presented aurally in either the preferred or non-preferred language, and subjects responded in the language in which the problem was presented.

The results were the same in both studies. Responses were faster in the preferred language than in the non-preferred language. However, the slope of the function relating RT to number of arithmetic operations was the same for the preferred and non-preferred languages, suggesting that the arithmetic fact retrieval process was the same in both conditions. As Marsh and Maki (1976) pointed out, this pattern of results is consistent with at least two interpretations. First, subjects may have translated the problems into the preferred language, and carried out the calculations on representations tied to this language. On this interpretation RT was slower in conditions requiring responses in the non-preferred language because the computed sums had to be translated from the preferred to the non-preferred language for output. The second interpretation states that subjects translated the problems into abstract language-independent representations, and then translated the answers into the preferred or non-preferred language for output. On this account RT was slower for responses in the non-preferred language because the translation of answers from abstract to language-specific form (and, in the McClain & Huang study, translation of problems from language-specific to abstract form) took longer for the non-preferred than for the preferred language. Thus, the Marsh and Maki (1976) and McClain and Huang (1982) results are equally consistent with format-specific and format-independent accounts of arithmetic fact retrieval.

Numerical comparison. When normal subjects judge which of two numbers is larger, responses are slower for numbers that are close in magnitude, such as 7 and 6, than for numbers that are not, such as 7 and 2 (e.g., Moyer & Landauer, 1967; for reviews, see Banks, 1977; Holender & Peereman, 1987; Moyer & Dumais, 1978). This numerical distance effect has been obtained for stimuli in a variety of forms, including arabic digits (e.g., Moyer & Landauer, 1967; Sekuler, Rubin, & Armstrong, 1971), written number words (e.g., Foltz, Poltrock, & Potts, 1984), patterns of dots (Buckley & Gillman, 1974), and Japanese kanji and kana numerals (Takahashi & Green, 1983). These results have been interpreted as evidence that regardless of the form in which stimuli are presented, performance on the task is mediated by abstract quantity representations that reflect magnitude relations among numbers.

However, some researchers (e.g., Besner & Coltheart, 1979; Takahashi & Green, 1983; Tzeng & Wang, 1983; Vaid, 1985; Vaid & Corina, 1989) have argued that performance in the numerical comparison task is influenced by the format of the stimuli, at least with respect to phenomena other than the numerical distance effect (e.g., the so-called "size congruity" effect; see Besner & Coltheart, 1979). These results have been taken to suggest that judgments of relative magnitude are made on the basis of codes tied to the form in which stimuli are presented, such as visual digit representations or phonological number-word representations (e.g., Campbell & Clark, 1988; Vaid, 1985). However, Holender and Peereman (1987), in a systematic review of stimulus format effects in numerical comparison, concluded that the available data are consistent with the assumption that stimuli in various forms are converted into a single form of internal representation.

At present the nature of the representations underlying performance in numerical comparison tasks remains an open issue. As in the case of semantic versus asemantic transcoding, progress in resolving the issue will probably depend not only upon additional empirical research, but also upon further development of the alternative theoretical perspectives. For instance, it will be important for proponents of the format-specific code position to provide a specific interpretation for the numerical distance effect and, more generally, to explain how relative magnitude judgments can be made on the basis of representations that do not directly specify quantity information, such as visual digit representations or phonological number-word representations.

Two new studies

My colleagues and I have recently carried out two studies of brain-damaged patients aimed specifically at testing the assumption of a modular cognitive architecture in which autonomous processing mechanisms communicate via a single form of internal numerical representation.

Patient PS: Manipulation of stimulus and response formats in arithmetic. I have already mentioned patient PS, who suffered from a deficit in retrieval of arithmetic facts. In testing PS on single-digit multiplication Sokol et al. (1991) manipulated the format of stimuli and responses. Problems were presented, and responses were elicited, in the form of arabic numerals (e.g., 3×4), written verbal numerals (e.g., *three times four*), and dots. In conditions involving dots stimuli, two thin manila envelopes were placed side by side with the multiplication symbol between them. In each envelope was a strip of paper containing a column of nine dots. A problem was presented by sliding each paper strip out of its envelope to expose the appropriate number of dots (e.g., 6 dots from the left envelope and 7 dots from the right envelope for 6×7). In conditions involving dots responses PS pulled strips of paper from envelopes to reveal dots representing tens and ones (e.g., 4 dots from a tens envelope and 2 dots from a ones envelope for the answer 42). PS was tested on all nine combinations of the three stimulus formats and the three response formats.

Results from several tasks indicated that PS was intact in comprehending and producing numbers in each of the tested formats. Hence, given the assumption that numerical inputs are converted to a single form of internal representation, the McCloskey et al. (1985) model predicts that the same pattern of impaired multiplication performance should be observed for all stimulus and response formats. If, in contrast, arithmetic fact retrieval involves format-specific representations and processes, then the pattern of impairment should presumably vary across stimulus and response formats. For example, one might expect to find that PS's fact retrieval impairment was limited to particular stimulus or response formats, or that she made different types of errors for different formats, or so forth.

In assessing the effects of the format manipulations Sokol et al. (1991) analyzed results for problems with operands in the range 1–9. (Across-format comparisons were not possible for 0's problems, because PS's performance on these problems changed drastically partway through the testing period, for reasons unrelated to the format manipulations; see pp. 149–150) PS's error rate on the 1–9's problems was clearly unaffected by stimulus or response format. For arabic, written verbal, and dots stimuli the error rates were 12%, 13%, and 13%, respectively; for arabic, written verbal, and dots responses error rates were 13%, 14%, and 12%, respectively.

Further, analyses in which PS's errors were classified into various types indicated that the distribution of errors across types was virtually identical for each stimulus and response format. Finally, Sokol et al. (1991) calculated the error rate for each individual problem (e.g., 6×4) for each stimulus and response format, and computed pairwise correlations between formats. The correlations were quite high, ranging from .83 to .90, indicating that PS tended to err on the same problems in each stimulus and response format.

This striking consistency in performance across the various forms of stimuli and responses strongly suggests that as stimulus and response formats were varied, the arithmetic fact retrieval process remained constant – that is, the same representations of problems were used to address the same representations of answers. Thus, the results support the assumption that arithmetic fact retrieval is mediated by internal numerical representations that are independent of the form in which stimuli are presented or responses are given.

The data do not, however, speak directly to the further claim that these representations are abstract quantity representations. For example, one might imagine that for all stimulus or response formats, problems are converted to phonological number-word representations (e.g., /sIks/ /taymz/ /eyt/), and that these representations are used to address phonological representations of answers (which may then be converted to other forms for output). Although we have no definitive evidence to offer on this point, the nature of PS's errors provides tentative support for the assumption that abstract quantity representations underlie the fact retrieval process. PS's fact retrieval errors were predominantly "operand" errors, in which the erroneous response is the correct answer to a problem sharing an operand with the stimulus problem (e.g., $7 \times 8 = 48$, in which the erroneous answer is correct for 6×8). In virtually all of the operand errors, the stimulus problem and the problem for which the answer was correct were close in magnitude with respect to the non-shared operand. For example, in $7 \times 8 = 48$ the stimulus problem (7×8) and the problem for which the response is correct (6×8) differ by 1 on the non-shared operand (7 vs. 6). In fully 95% of PS's operand errors (172/182) the non-shared operand was within ± 2 of the correct operand. Thus, PS's erroneous responses were correct for problems with operands numerically (as opposed to phonologically or visually) similar to the operands in the stimulus problem. This *operand distance effect* is also observed in the errors made by normal subjects (e.g., Campbell & Graham, 1985; Miller, Perlmutter, & Keating, 1984). Although several interpretations may be offered for the effect (see Sokol et al., 1991; McCloskey et al., 1991), it is at least consistent with the assumption that arithmetic fact retrieval is mediated by abstract quantity representations.

Patient RH: Multiple deficits in numeral processing. Macaruso, McCloskey, and Aliminosa (in press) studied lexical processing of numerals in patient RH, a 42-year-old man with a B.A. in chemistry who underwent surgery for removal of a left temporal–parietal brain tumor. RH was tested with 12 transcoding tasks, comprising all translations among numbers in the form of arabic numerals, written verbal numerals, spoken verbal numerals, and dots. For example, in the arabic-to-spoken-verbal transcoding task RH read aloud arabic numerals (e.g., stimulus 84, correct response "eighty-four"). Stimuli were limited to numbers in the range 0–99 so that lexical processing (i.e., processing of individual elements of a numeral) could be studied without complications introduced by RH's severe deficits in syntactic processing of numerals. The results for the 12 tasks are shown in Table 1, in the column labelled "Actual error percentage."

According to the McCloskey et al. (1985) model, each transcoding task involves a comprehension process that converts the stimulus into an abstract internal representation, and a production process that transforms the internal representation into the appropriate form for output. For example, the arabic-to-written-verbal transcoding task requires arabic numeral comprehension processes, and the graphemic processing mechanisms within the verbal numeral production component.[5] From the assumption of autonomous comprehension and production processes communicating via a single form of abstract internal representation, it follows that the error rate on any given task should be a joint function of the extent of damage to the comprehension process, and the extent of damage to the production process. Thus, it should be possible to explain the pattern of performance across the 12 tasks by specifying the extent of impairment to each of the 8 underlying comprehension and production processes. On the other hand, if the assumption of a modular functional architecture is incorrect (as Campbell & Clark, 1988, and Clark & Campbell, 1991 suggest), one would not expect RH's performance to be explicable by reference to independent comprehension and production processes, each of which is implicated in multiple tasks.

Table 1. *Actual and expected error percentages on the 12 transcoding tasks for patient RH*

		Error percentage	
Stimulus form	Response form	Actual	Expected
Arabic	Spoken verbal	25.5	24.8
Arabic	Written verbal	88.0	88.6
Arabic	Dots	3.0	3.1
Spoken verbal	Arabic	21.0	19.7
Spoken verbal	Written verbal	89.5	90.3
Spoken verbal	Dots	16.5	17.4
Written verbal	Arabic	6.5	10.2
Written verbal	Spoken verbal	33.5	28.3
Written verbal	Dots	7.5	7.6
Dots	Arabic	11.5	8.2
Dots	Spoken verbal	21.5	26.7
Dots	Written verbal	90.0	88.9

[5]Macaruso et al. (in press) assumed that the tasks involving dots make use of dots comprehension and production processes developed on the basis of experience with the dots format.

Macaruso et al. (in press) attempted to fit RH's performance pattern quantitatively to the McCloskey et al. (1985) model. Applying an iterative gradient descent algorithm to a set of equations expressing the model's assumptions about the comprehension and production processes contributing to each task, they first derived from the error rates on the 12 tasks estimates of the extent of damage to each of the underlying processes. These estimates are presented in Table 2. The value for a process represents an estimate of the probability of error for that process on any given trial. For example, the .174 estimated error probability for spoken verbal numeral comprehension indicates that this process is expected to generate an incorrect semantic representation for a dictated numeral on 17.4% of trials (for stimuli in the 0–99 range).

The critical question for the McCloskey et al. (1985) model was: to what extent can the pattern of error rates across the 12 transcoding tasks be interpreted in terms of the estimated error probabilities for the hypothesized underlying comprehension and production processes? To answer this question, Macaruso et al. (in press) computed for each task the performance expected on the basis of the estimated error probabilities. Consider, for example, the Arabic-to-spoken-verbal task. Assuming that comprehension and production errors represent independent events, the probability of error in the task is given by $P_{AC} + P_{SVP} - P_{AC}P_{SVP}$, where P_{AC} is the probability of error in the arabic comprehension process, P_{SVP} is the probability of error in the spoken verbal production process, and $P_{AC}P_{SVP}$ is the product of these two probabilities. Carrying out the computation using the estimated error probabilities of .031 for arabic numeral comprehension and .224 for spoken verbal numeral production, one arrives at an expected error rate for the arabic-to-spoken verbal transcoding task of .248 (i.e., 24.8%), which accords well with the observed error rate of .255(25.5%).

The column labeled "Expected error percentage" in Table 1 presents the expected error rates for each of the 12 tasks. It is apparent from the table that the observed and expected error rates corresponded very closely; for no task were the

Table 2. *Derived error probabilities for patient RH for each comprehension and production process*

Process		Error probability
Comprehension	Arabic	.031
	Spoken verbal	.174
	Written verbal	.076
	Dots	.056
Production	Arabic	.028
	Spoken verbal	.224
	Written verbal	.882
	Dots	.000

two values reliably different.[6] Thus, RH's pattern of performance across the 12 transcoding tasks may be explained in terms of damage to the underlying numeral comprehension and production processes posited by the McCloskey et al. (1985) model.

Examination of error patterns across tasks provided further support for the claim that RH suffered from damage to independent numeral comprehension and production processes. For example, on the three tasks requiring written verbal responses RH showed a virtually identical pattern across tasks in error rates and types of errors for individual words. These results support the assumption that the same disrupted numeral production process was employed in all three tasks.

Finally, the error data provided evidence for the involvement of semantic representations in the transcoding tasks. For each of the tasks showing substantial impairment RH displayed a very strong tendency to produce incorrect digits or words similar in magnitude (and not visually or phonologically similar) to the correct digits or words. For example, in tasks involving production of spoken verbal numerals, the incorrect number words produced by RH were consistently close in magnitude to the correct words (e.g., *eighty-three* read aloud as "ninety-three" and 47 read as "forty-six"). This result supports the assumption of a functional architecture in which comprehension and production processes communicate via abstract semantic representations.

Summary

The preceding discussion illustrates both that recent research has confronted fundamental issues concerning the nature of cognitive numerical processing mechanisms, and that these issues remain very much open. In particular, the extent to which numerical processing mechanisms are modular, and the forms of numerical representation implicated in central processing of numbers, are likely to be debated for some time to come. Regardless of the ultimate resolution of these issues, the most productive approach would appear to be that of beginning with a relatively simple and constrained model (whether the McCloskey et al. model, or some alternative model yet to be developed), adding processing

[6]The close correspondence between actual and expected error rates is not a trivial result. Because each hypothesized numeral comprehension and production process is assumed to be involved in three of the 12 tasks, each estimated error probability must contribute to explaining three different task error rates. For example, the estimated error probability for written verbal numeral production must contribute to interpreting the error rates for arabic-to-written-verbal, spoken-verbal-to-written-verbal, and dots-to-written-verbal transcoding. As a consequence, it is not the case that any arbitrary pattern of error rates could be fit to the model. To demonstrate that the model could not accommodate any arbitrary pattern of results, Macaruso et al. (in press) randomly assigned the 12 observed error rates to the 12 tasks, and then attempted to fit these arbitrary patterns to the model. In 498 of 500 different random assignments "actual" and expected performance differed reliably for at least one task; for 474 of the random assignments, at least 9 of the 12 tasks showed reliable actual/expected differences.

mechanisms or otherwise relaxing constraints only when the original set of assumptions proves inadequate. In contrast, the wholesale abandonment of constraints exemplified by the encoding complex view in its current form (Campbell & Clark, 1988; Clark & Campbell, 1991) seems unlikely to prove fruitful.

INTERNAL STRUCTURE AND FUNCTIONING OF PROCESSING COMPONENTS

In addition to exploring the general functional architecture of the cognitive numeral processing and calculation mechanisms, recent research on dyscalculia has probed the internal structure of particular processing mechanisms within this general architecture. For the most part attention has focused on mechanisms for verbal numeral production, and arithmetic fact retrieval. In the following sections I consider these two topics in turn.

Cognitive processes in production of verbal numerals

My colleagues and I have recently studied several patients with impairments in production of verbal numerals (McCloskey et al., 1986; McCloskey, Sokol, Goodman-Schulman, & Caramazza, 1990; Sokol & McCloskey, 1988), with the aim of characterizing the representations and processes underlying verbal numeral production. On the basis of the McCloskey et al. (1985) model we assumed that the verbal numeral production process takes as input an abstract semantic representation of a number, and generates as output a sequence of phonological (spoken output) or graphemic (written output) number word representations. In this section I review the results of our studies, and also consider a recent report by Cohen and Dehaene (1991) that raises questions about some of our conclusions.

I will discuss verbal numeral production primarily in terms of English verbal numerals. Most aspects of the production process are presumably common to languages with substantially similar verbal numeral systems (e.g., French), and fundamental aspects of the process may well be shared still more broadly. Nevertheless, it is important to bear in mind that the lexical and syntactic structure of verbal numerals varies considerably across languages, and this variation may be reflected in the representations and processes underlying verbal numeral production. (For further discussion see McCloskey et al., 1986; Deloche & Seron, 1987; and Cohen & Dehaene, 1991.)

Linguistic structure of English verbal numerals

A brief examination of the linguistic structure of verbal numerals may help to make clear the nature of the computations a verbal numeral production process must carry out. In the English verbal numeral system, the "ones" words *one* through *nine* represent the basic quantities {1} through {9}. The "tens" words (*twenty*, *thirty*, . . . ,*ninety*) represent basic quantities times ten. Numbers including both tens and ones are represented by combining the appropriate tens word with the appropriate ones word (e.g., *thirty-six*). The single exception to this rule is that numbers made up of one ten and some ones are represented by a special set of "teens" words. For instance, the number consisting of one ten and six ones is represented by the word *sixteen*.

Numbers larger than ninety-nine are represented by associating ones, teens, and tens words with multiplier words such as *hundred*, *thousand*, and *million*. The basic unit in a verbal numeral is the sequence [ONES hundred TENS ONES] (e.g., *six hundred thirty-seven*). The word *hundred* in this sequence serves as a multiplier for the preceding ones word, multiplying its value by one hundred. In any specific instantiation of the [ONES hundred TENS ONES] unit, some elements may be null, as in *six hundred seven* (in which there is no tens word). Further, the sequence TENS ONES may be replaced by TEENS, as in *six hundred seventeen*.

Whereas hundred acts as a multiplier for a single word, the multipliers *thousand*, *million*, and so forth, multiply entire units. For example, in the numeral *six hundred thirty-seven thousand*, *thousand* multiplies the entire unit *six hundred thirty-seven*.

A complete number is represented simply by assembling the basic units, with their associated multipliers, in order of decreasing magnitude. Thus, the number 634,546,321 is represented by arranging the following units in the order shown:

((six) hundred thirty-four) million ((five) hundred forty-six) thousand
((three) hundred twenty-one)

The parentheses show the scope of the multipliers: each multiplier applies to the expression enclosed by the set of parentheses to its immediate left. Thus, in the millions unit, hundred multiplies six, and million multiplies the entire unit.

Results from several patients suggest that this rich lexical and syntactic structure is reflected in the cognitive mechanisms underlying production of verbal numerals.

Patient HY

McCloskey et al. (1986) probed production of spoken verbal numerals in patient HY, a 69-year-old right-handed man who sustained left temporal–parietal damage as a consequence of a CVA. Over a period of several months HY read aloud nearly 5000 1- to 7-digit arabic numerals, with an error rate of 14%. Examples of

Table 3. *Examples of patient HY's errors in reading aloud arabic numerals*

Stimulus	Response
1	*five*
17	*thirteen*
29	*forty*-nine
317	three hundred *fourteen*
14,840	*sixteen* thousand eight hundred forty
940,711	nine hundred *twenty* thousand *five* hundred eleven

his errors are presented in Table 3. The vast majority of errors were lexical substitutions in which an incorrect number word was substituted for the correct word. For example, in the response "three hundred fourteen" to the stimulus 317, the incorrect word *fourteen* was produced in place of the correct word *seventeen*.

According to the McCloskey et al. (1985) model, reading aloud an arabic numeral (e.g., 20,012) involves first an arabic numeral comprehension process that translates the arabic stimulus into an internal semantic representation (e.g., {2}10EXP4, {1}10EXP1, {2}10EXP0). This representation then serves as input to a verbal numeral production process, which generates as output a sequence of phonological number-word representations (e.g., /twenty/ /thousand/ /twelve/). (For convenience, I will henceforth use words enclosed in slashes to indicate phonological representations.)

Results from several numeral–processing tasks suggested that HY's errors in reading aloud arabic numerals reflected a deficit in production of spoken verbal numerals, and not an impairment in comprehension of arabic numerals: in tasks requiring arabic numeral comprehension HY performed well, whereas in tasks requiring spoken production of verbal numerals he made lexical substitution errors (McCloskey et al., 1986).

McCloskey et al. (1986) conducted an analysis in which HY's responses in reading aloud arabic numerals were compared word-by-word with the corresponding correct responses. For example, HY responded "five thousand four hundred seventy" to the stimulus 5450. Comparing this response to the correct response "five thousand four hundred fifty" reveals that the words *five*, *thousand*, *four*, and *hundred* were produced correctly, but that *seventy* was substituted for the correct word *fifty*. A confusion matrix tabulating the results of this analysis for the ones, teens, and tens words is shown in Figure 3. The rows of the matrix represent the correct response words, and the columns represent the words produced by HY. For example, the row for the word *two* indicates that when the correct response word was *two*, HY never said "zero," but said "one" 10 times, "two" 649 times, "three" once, and so forth. Note that the unit of analysis is the individual word, and not the response as a whole.

HY's errors were not evenly distributed across the possible incorrect response words. Rather, the errors fell into three distinct clusters. When the correct word

	zero	one	two	three	four	five	six	seven	eight	nine	ten	eleven	twelve	thirteen	fourteen	fifteen	sixteen	seventeen	eighteen	nineteen	twenty	thirty	forty	fifty	sixty	seventy	eighty	ninety
zero	64																											
one		615	15		1	5		2	3	2	1																	
two		10	649	1	7	1	14	1	2	1	1																	
three			14	618	2	27	8	10	3	1		1										2						
four		1	1	2	653	3	7	7	8	3																		
five		1	2	4	9	646	3	17	6	5	1					2												
six			8	3	7	4	649	1	17	1															1			
seven		7	1	1	12	10	2	624		13																5		
eight		1	3	2	8	9	17	7	607	2																		
nine			1		2	2	4	12	1	615	2																	
ten		1	1					1			181				2		1			1	1							
eleven										1	1	158		1	1	1		2										
twelve							1				1		154	1	4	1	30		1		1							
thirteen												2		172		1	3	7										
fourteen													1		164		2	3	1	1								
fifteen											4	2		1	1	129	3	8										
sixteen													1				151	1	2									
seventeen												1		1	4	1		135	2	2								
eighteen													1				11	1	137	2								
nineteen											1	1			1		4	5	2	150								1
twenty			1				2				1		1								263	4	4	3	9		3	
thirty						1		1	1					2							2	277		8	3	3	3	
forty					3										3						7	2	277	1		5	2	2
fifty						1					4					1					3	6	4	266	5	7		
sixty																	1				7	1	1	2	277		3	
seventy								4										1					7	4	2	263	2	1
eighty																			1		3	1	6	1	10	4	247	3
ninety										1											1		1	5	2	10	3	269

Figure 3. *Number-word confusion matrix for patient HY.*

was a ones word (i.e., between *one* and *nine*), nearly all of the incorrect words produced by HY were also ones words. When the correct word was a teens word (i.e., between *ten* and *nineteen*), nearly all of HY's errors were also teens words. Finally, when the correct word was a tens word (i.e., *twenty*, *thirty*, *forty*,..., *ninety*), HY's errors were almost always tens words. Over 90% of HY's errors (665 of 720) fell into one of these three clusters.

A model of spoken verbal numeral production

With HY's pattern of lexical substitution errors as a principal motivation, McCloskey et al. (1986) proposed a model of the verbal numeral production process. The major assumptions of the model concern the representation of number words in the phonological output lexicon (i.e., the store of phonological word representations underlying the production of spoken language), and the processes that compute from a semantic representation of a number the sequence of number words to be retrieved from the lexicon.

The phonological output lexicon. The model assumes that within the phonological output lexicon the representations of the basic number words (i.e., one, two,...,eighty, ninety) are partitioned into three functionally distinct classes, as shown in Table 4. The ONES class contains phonological representations of the words *one* through *nine*, the TEENS class contains phonological representations of the words *ten* through *nineteen*, and the TENS class contains phonological representations of the words *twenty*, *thirty*, and so forth, up to *ninety*. Retrieval of a phonological representation thus involves specifying the appropriate lexical class (e.g., TENS), and the appropriate position within the class (e.g., the position for the quantity {5}). For example, the phonological representation /fifty/ would be addressed by the class/position-within-class specification TENS:{5}, and TEENS:{2} would specify /twelve/. The output lexicon is also assumed to include a MULTIPLIER class containing phonological representations of the multiplier words *hundred*, *thousand*, *million*, and so forth.

The numeral production process. McCloskey et al. (1986) assumed that when the verbal numeral production process receives a semantic representation of a number, the largest power of ten in the number is identified, and on this basis a "syntactic frame" is generated. Consider, for example, the semantic representation {3}10EXP4, {7}10EXP2, {9}10EXP1, corresponding to the arabic stimulus 30,790. From this input, the following frame would be generated:

[TENS:__ ONES:__] MULT:T [[ONES:__] MULT:H TENS:__ ONES:__]
"10EXP4" "10EXP3" "10EXP2" "10EXP1" "10EXP0"

Table 4. *Number-lexical classes*

	Lexical class		
Quantity	Ones	Teens	Tens
{0}	–	ten	–
{1}	one	eleven	–
{2}	two	twelve	twenty
{3}	three	thirteen	thirty
{4}	four	fourteen	forty
{5}	five	fifteen	fifty
{6}	six	sixteen	sixty
{7}	seven	seventeen	seventy
{8}	eight	eighteen	eighty
{9}	nine	nineteen	ninety

This frame specifies, among other things, that for numbers in which 10EXP4 is the largest power of ten, the canonical verbal form is a TENS word, followed by a ONES word, followed by the MULTIPLIER *thousand*, followed by a ONES word, and so forth.

After the syntactic frame is generated, each basic quantity in the input semantic representation is assigned to the appropriate slot in the frame. This filling of the frame is guided by the labels beneath each slot. For example, the label under the leftmost slot specifies that this slot should be filled with the quantity associated with 10EXP4 (i.e., {3}). Thus, for 30,790 the filled frame takes the following form:

[TENS:{3} ONES:__] MULT:T [[ONES: {7}] MULT:H TENS: {9} ONES:__]
"10EXP4" "10EXP3" "10EXP2" "10EXP1" "10EXP0"

The filled syntactic frame represents a plan for production of the sequence of words comprising the verbal numeral. The individual elements in the frame (e.g., TENS:{3}) are abstract lexical–semantic representations of the to-be-produced words. These abstract representations are used to address phonological representations in the output lexicon. For each filled slot the class label (e.g., TENS) indicates the number-lexical class, and the quantity representation (e.g., {3}) identifies the position within class. Thus, for example, the leftmost slot specifies retrieval of the phonological representation /thirty/. An empty slot, such as the "10EXP3" slot in the frame shown above, indicates that no phonological representation should be retrieved. Multiplier words are specified in terms of the MULTIPLIER class (abbreviated MULT in the present examples) and the particular item within that class. Thus, MULT:T and MULT:H represent instructions for retrieval of the phonological representations /thousand/ and /hundred/, respectively. The final step in the production process is therefore the retrieval of the phonological representations specified by the filled syntactic frame. In the present example, the retrieval process would yield the sequence /thirty/ /thousand/ /seven/ /hundred/ /ninety/.

The lexical retrieval process is assumed to unfold as described, unless a {1} is encountered in a TENS-class slot. In this event, a special "teens" procedure is invoked. This procedure does not retrieve a phonological representation from the TENS class, but instead proceeds to the next slot and pairs the quantity value in that slot with the class label TEENS, in order to address a phonological representation in the TEENS class. For example, for the filled frame shown below the teens procedure would generate TEENS:{3}, so that the lexical retrieval stage would yield the sequence /six/ /hundred/ /thirteen/:

[[ONES:{6}] MULT:H TENS:{1} ONES:{3}]
"10EXP2" "10EXP1" "10EXP0"

If the slot that specifies the position within the TEENS class is empty (as it would be for 610), then the teens procedure is assumed to generate the specification TEENS:{0}, leading to retrieval of /ten/.

Finally, one additional assumption about multipliers and empty slots is needed. The brackets in the syntactic frame indicate the slots to which the multipliers apply. If all slots within the scope of a multiplier are empty, then the multiplier word is omitted. Consider, for example, the filled frame for the number 30,079:

[TENS:{3} ONES:_] MULT:T [[ONES:_] MULT:H TENS:{7} ONES:{9}]
"10EXP4" "10EXP3" "10EXP2" "10EXP1" "10EXP0"

The multiplier hundred applies only to the ONES slot to its immediate left. Because this slot is empty, the phonological form /hundred/ is not retrieved, and the retrieval process yields the sequence /thirty/ /thousand/ /seventy/ /nine/.

Thus, the McCloskey et al. (1986) model assumes that production of verbal numerals involves a progression through three levels of representation. The input to the process is a semantic representation of a number. This representation is converted to an abstract representation of the corresponding verbal numeral, a representation comprising lexical–semantic representations of the individual number words as well as syntactic information about the word sequence as a whole (i.e., information about word order, and scope of multipliers). Finally, the abstract numeral representation is converted to a sequence of phonological number-word representations.

Application of the Model to HY's performance

The McCloskey et al. (1986) model provides a straightforward account of HY's error pattern in terms of a deficit affecting retrieval of phonological number-word representations from the phonological output lexicon. McCloskey et al. (1986) argued that given a specification of a to-be-retrieved number word, HY was largely intact in accessing the correct number-lexical class, but was impaired in accessing the appropriate position within class. For example, given the specification TENS:{7}, the lexical retrieval process would almost always access the TENS class. However, the retrieval process might access the wrong position within the TENS class, resulting in the substitution of an incorrect tens word (e.g., *forty*, *sixty*) for the correct word *seventy*. A deficit of this sort should lead to within-class lexical substitution errors (i.e., errors in which the incorrect words are from the same number-lexical class as the correct words). This is, of course, just what the confusion matrix in Figure 3 shows – HY substituted ones words for ones words, teens words for teens words, and tens words for tens words.

Patient JE

McCloskey et al. (1990) reported results from a patient whose errors in reading arabic numerals were quite different from those of HY. Patient JE, a 48-year-old man who suffered a left hemisphere CVA, read aloud 848 arabic numerals of 1–5 digits, with an error rate of 6.4%. Approximately one-fourth of JE's errors were within-class lexical substitutions (e.g., stimulus 5097, response "five thousand ninety-*five*"). The remaining three-fourths were "quantity shift" errors, in which a quantity associated with one power of ten in the arabic stimulus was associated with another power of ten in the verbal response. For example, JE read aloud 8900 as "eight thousand ninety." Thus, the nine hundreds in the stimulus became nine tens in the response. Other examples are presented in Table 5. Most of the quantity shift errors were not interpretable as lexical substitutions in which an incorrect word was retrieved in place of a correct word, because the incorrect responses usually contained a different number of words than the corresponding correct responses. For example, the correct response to the stimulus 5012 contains three words, but JE's response – "five thousand one hundred two" – included five.

On tests of arabic numeral comprehension JE's performance was perfect, suggesting that his errors in reading aloud arabic numerals stemmed from disruption of the verbal numeral production process. McCloskey et al. (1990) suggested in particular that JE's quantity shift errors reflected a deficit in generating and filling syntactic frames for verbal numerals. More specifically, they suggested that JE was impaired in using the power-of-ten specifications in semantic representations of numbers to generate the appropriate syntactic frame, and to assign quantity representations to the appropriate slots in the frame.

Consider, for example, the stimulus 5012, to which JE responded "five thousand one hundred two." For this stimulus, the arabic numeral comprehension process should generate the semantic representation {5}10EXP3, {1}10EXP1, {2}10EXP0, and from this semantic representation the verbal numeral production process should create the following syntactic frame:

$$\begin{array}{llllll} [\ \text{ONES:}\underline{\{5\}}\ ] & \text{MULT:T}\ [& [\ \text{ONES:}\underline{\ \ }\ ] & \text{MULT:H} & \text{TENS:}\underline{\{1\}} & \text{ONES:}\underline{\{2\}}\ ] \\ \text{“10EXP3”} & & \text{“10EXP2”} & & \text{“10EXP1”} & \text{“10EXP0”} \end{array}$$

Suppose, however, that in the process of filling the syntactic frame the 10EXP1

Table 5. *Examples of patient JE's quantity shift errors in reading aloud arabic numerals*

Stimulus	Response
5012	five thousand *one hundred two*
7900	seven thousand *ninety*
1200	one thousand *twenty*
2070	*twenty thousand* seventy
6003	*six hundred* three
45,010	forty-five thousand *one hundred*

specification associated with the quantity {1} in the semantic representation were misinterpreted as 10EXP2. The result would be assignment of the {1} representation not to the correct "10EXP1" slot in the syntactic frame, but rather to the "10EXP2" slot:

[ONES:{5}] MULT:T [[ONES:{1}] MULT:H TENS:__ ONES:{2}]
"10EXP3" "10EXP2" "10EXP1" "10EXP0"

This error in filling the syntactic frame would lead to the incorrect response "five thousand one hundred two."

Patient JS

Sokol and McCloskey (1988) studied a patient whose performance in written and spoken production of verbal numerals supported the assumptions of the McCloskey et al. (1986) model concerning the architecture of the verbal numeral production mechanisms. The model assumes that the syntactic frame constructed as the first step in the production process specifies the to-be-produced number words in abstract lexical–semantic form (e.g., TENS:{4}, ONES:{8}). At this abstract planning level, neither the representations nor the processes that compute these representations are tied specifically to the spoken production mode. Consequently, the model predicts that this initial planning stage is common to spoken and written production of verbal numerals.

Written and spoken production should diverge, however, at the lexical retrieval stage. Spoken production requires retrieval of phonological number-word representations from a phonological output lexicon, whereas written production requires retrieval of graphemic representations from a graphemic output lexicon. Consistent with these assumptions patient JS exhibited both a syntactic association and a lexical dissociation in spoken versus written verbal numeral production.

JS was presented with arabic numerals of 1–9 digits, and asked either to read the numeral aloud (spoken production task), or to write the numeral in word form (written production task). Approximately 250 stimuli were presented in each task. JS's overall error rate was 27% in the spoken production task, and 18% in the written production task. On tests of arabic numeral comprehension, however, his performance was excellent, suggesting that the errors in the spoken and written verbal numeral production tasks reflected a deficit in the production of the verbal responses, and not in the comprehension of the arabic stimuli.

Spoken production task. In reading aloud arabic numerals JS made two distinct types of errors. For 13% of the stimuli, he made lexical substitutions, nearly all of which were within-class errors (e.g., stimulus 309, response "*six* hundred nine"). Sokol and McCloskey (1988) interpreted these errors as reflecting a mild deficit in retrieval of phonological number word representations from the phonological output lexicon.

In addition to the lexical substitution errors, JS made syntactic errors for 17% of the stimuli. These errors involved incorrect arrangements of multiplier words in the spoken response. For example, JS read aloud 146,359 as "one hundred *thousand* forty-six three hundred fifty-nine." The syntactic errors occurred only for 6- through 9-digit stimuli, and in fact were limited to the thousands or millions unit of the response (i.e., the unit correctly produced as ONES hundred TENS ONES thousand, as in *five hundred eighty-nine thousand*, or ONES hundred TENS ONES million, as in *seven hundred fifty-nine million*). Sokol and McCloskey (1988) noted that these units are syntactically complex because they involve multiplication of basic number-word values by more than one multiplier (e.g., in *five hundred eighty-nine thousand*, *hundred* multiplies *five*, and *thousand* multiplies the entire unit). They suggested that JS's syntactic errors stemmed from a deficit in placement of multiplier word specifications in syntactic frames for syntactically complex units.

Written production task. According to the McCloskey et al. (1986) model, construction of a syntactic frame should be common to spoken and written production of verbal numerals. Consistent with this assumption, JS presented with a pattern of syntactic errors in the written production task that was virtually identical to his pattern in the spoken production task. Syntactic errors in written production occurred for 18% of the stimuli (compared to 17% in the spoken task), and as in spoken production involved only the thousands or millions units in responses to 6- through 9-digit stimuli. Finally, the syntactic errors in the written production task were of the same sorts as in the spoken production task, as the examples in Table 6 illustrate. Sokol and McCloskey (1988) took the close similarity of syntactic error patterns across the two tasks as support for the assumption of a syntactic frame generation process common to spoken and written production of verbal numbers.

Table 6. *Examples of patient JS's syntactic errors in spoken and written production of verbal numerals*

Stimulus	Response
Spoken production task	
407,013	*four hundred thousand seven*, thirteen
4,258,631	four million, *two thousand fifty-eight*, six hundred thirty-one
349,258,107	*three forty-nine million*, *two fifty-eight thousand*, one hundred seven
Written production task	
358,916	*three hundred thousand fifty-eight* nine hundred sixteen
209,712	*two thousand nine* seven hundred twelve
230,561,317	*two thirty million five sixty-one thousand* three hundred seventeen

JS's performance also supported the assumption that spoken and written production diverge at the lexical retrieval stage. Whereas JS made lexical substitution errors for 13% of the stimuli in the spoken production task, he made no lexical errors in the written production task. Sokol and McCloskey (1988) interpreted this dissociation by assuming that JS was impaired in retrieval of phonological number-word representations from the phonological output lexicon, but not in retrieval of graphemic number-word representations from the graphemic output lexicon.

Patient YM

The findings of the studies reviewed thus far are consistent with the McCloskey et al. (1986) model of verbal numeral production (although see Campbell & Clark, 1988, for a different opinion with respect to patient HY). However, a recent study by Cohen and Dehaene (1991) complicates the picture to some extent. Cohen and Dehaene studied a French patient, YM, a 58-year-old man suffering from a left temporal–parietal tumor. In reading aloud arabic numerals, YM made substitution errors, such as 3 read as "neuf" (nine). According to the McCloskey et al. (1986) model, such errors could result from deficits in arabic numeral comprehension and/or verbal numeral production – that is, impairments in translating arabic stimulus numerals into semantic representations of numbers, and/or in translating semantic representations into sequences of phonological number-word representations.

Results from other tasks suggested an impairment in verbal numeral production. Written addition problems involving 1- and 2-digit numbers were presented with correct or incorrect answers. Although YM erred in reading aloud 15 of the 25 problems, he made only 3 errors in indicating whether the answer to a problem was correct or incorrect. The reasonably good verification performance implies that the reading errors were not entirely due to errors in comprehension of the arabic numerals in the problems, and hence suggests a deficit in production of spoken verbal numerals.

However, other findings are not readily interpretable in terms of a deficit in translating semantic representations of numbers into a sequence of phonological number-word representations. The incorrect digits in arabic transcriptions of YM's number-reading responses tended to be visually similar to the corresponding correct digits (e.g., stimulus 233, response 733). Further, when the digits in a stimulus number were arrayed in the standard left-to-right fashion, YM was much more likely to err on the leftmost digit than on subsequent digits. However, when the digits were arranged vertically, error rate did not differ as a function of position. These results suggest a deficit at a level of representation involving spatially arrayed visual representations of digits.

Within the framework of the McCloskey et al. (1986) model the effects of visual/spatial factors suggest that in addition to a verbal numeral production deficit, YM also had a deficit in comprehension of arabic numerals (e.g., a relatively peripheral deficit affecting the translation of digits, especially the leftmost digit, into abstract quantity representations). Some findings from other tasks, although not definitive, are consistent with this interpretation. As mentioned above, YM made some errors in verifying addition problems (3 of 25, or 12%); further, when asked to produce written answers to addition problems, YM erred on 6 of 25 problems (24%). These errors, which are comparable in frequency to YM's errors in reading aloud 1- and 2-digit numbers (13% and 17% error rates, respectively), could reflect impairment in comprehension of arabic stimulus numbers (although a calculation deficit could also produce such errors).[7]

One result, however, argues against a deficit in arabic numeral comprehension: when presented with 220 pairs of 1- to 4-digit arabic numerals and asked to judge which number in each pair was larger, YM was 100% correct. (When asked to read aloud 88 of the pairs, YM made 38 errors, and six of these errors resulted in a reversal of the relative magnitudes of the two numbers.)

From YM's pattern of performance across the various tasks, Cohen and Dehaene (1991) suggested that the McCloskey et al. (1986) model should be modified. Whereas the McCloskey et al. model assumes that verbal numeral production processes operate upon an abstract semantic representation of a number, Cohen and Dehaene argued that in reading arabic numerals aloud spatially arrayed digit representations are used as a basis for generating a syntactic frame for the to-be-produced verbal numeral. Then, for each digit that specifies the position-within-class of a ones, teens, or tens words in the numeral, the "identity" of the digit is retrieved by a digit identity retrieval process. For example, in reading aloud the arabic numeral 304, digit representations (3-0-4) would be used to generate a syntactic frame. The digit identity retrieval process would then insert the identities of the digits 3 and 4 into the syntactic frame. Finally, these digit identities would be used in combination with the number-lexical class representations (e.g., TENS) to retrieve the appropriate phonological number-word representations. Given these assumptions, Cohen and Dehaene (1991) argue, YM's deficit can be localized to the digit identity retrieval process.

Although the Cohen and Dehaene (1991) arguments are interesting, two important aspects of their theoretical proposal are not clearly articulated. First, in discussing digit identity retrieval Cohen and Dehaene do not explain what they mean by digit "identities." Thus, it is unclear just what sort of representation the digit identity retrieval process is presumed to retrieve. Second, Cohen and

[7]Results from another task suggest that a deficit in arabic numeral production was probably not responsible for the errors in producing written responses to addition problems.

Dehaene do not indicate whether their model is intended to apply to verbal numeral production in general, or only to arabic-to-spoken-verbal transcoding. Is it assumed that any task requiring production of verbal numerals implicates the production process proposed for reading of Arabic numerals (i.e., a process that takes as its input a sequence of visual digit representations)? Or is the assumption instead that are there several (at least partially) distinct verbal numeral production processes, each of which operates upon a different form of input? For example, in producing spoken responses to questions like "How tall is the average English woman?", is an answer first generated in the form of an abstract semantic representation, then converted to a sequence of visual digit representations, and finally operated upon by the verbal numeral production process described by Cohen and Dehaene? Or is the initially generated abstract representation taken as input by a verbal numeral production process distinct from the process involved in reading aloud arabic numerals?

In the absence of more explicit assumptions about the scope of the model, and the nature of digit identity representations, the Cohen and Dehaene proposal is difficult to evaluate with respect to theoretical motivation and empirical adequacy. For example, on the interpretation that YM's errors in reading aloud arabic numerals reflected a deficit in digit identity retrieval, what should we expect with regard to performance on other tasks? Should a digit identity retrieval deficit lead to impairment in judging which of two arabic numerals is larger, or in production of written verbal numerals from arabic stimuli, or so forth? These questions are difficult to answer, because there is no clear basis for assessing whether a task should require digit identity retrieval.

At the empirical level, Cohen and Dehaene's (1991) arguments are also not as strong as they might be. Nearly all of the findings from patient YM can be accommodated within the framework of the McCloskey et al. (1986) model by assuming deficits in both arabic numeral comprehension and verbal numeral production. The only result raising difficulties for this interpretation is YM's perfect performance in the numerical comparison task (i.e., the task in which he judged which of two arabic numerals was larger), because this finding suggests intact comprehension of arabic numerals. However, the numerical comparison results, while certainly troublesome, are less than compelling. Although the interpretation based on the McCloskey et al. (1986) model predicts some errors in numerical comparison, this interpretation does not predict that numerical comparison errors should have occurred as often as reading-aloud errors that reversed the magnitude of the numbers in a pair. From the perspective of the McCloskey et al. (1986) model, not all of the reading-aloud errors were due to an arabic numeral comprehension deficit; some of these errors reflected a deficit in verbal numeral production. Thus, although Cohen and Dehaene (1991) reported that 6 of the errors YM made in reading aloud 88 of the numerical comparison stimuli reversed the relative magnitude of the stimuli, it does not follow from the

McCloskey et al. (1986) model that numerical comparison errors should have occurred at such a high rate. Given that no other evidence of intact arabic numeral comprehension was presented, the numerical comparison results do not provide a strong basis for excluding the possibility of an arabic numeral comprehension deficit. This is especially true in light of the fact that YM's substantial error rates in arithmetic tasks not requiring spoken responses are consistent with such a deficit.

The Cohen and Dehaene (1991) study raises at a more specific level the issues discussed in earlier sections concerning the functional architecture of numerical processing mechanisms, and the forms of representation implicated in various aspects of processing. Thus, the study highlights once again the centrality of these issues in current cognitive neuropsychological research on numerical processing.

Facts, rules, and procedures in arithmetic

The impaired performance of brain-damaged patients has also been used to explore the internal structure of processing mechanisms underlying basic arithmetic performance. Over the past several years my colleagues and I have carried out a number of single-case studies involving patients with deficits in arithmetic fact retrieval (McCloskey, Aliminosa, & Sokol, 1991; Sokol et al., 1991; Sokol & McCloskey, 1991; Sokol, McCloskey, & Cohen, 1989). In this section I survey some of the major findings from 10 patients tested extensively in single- and multi-digit multiplication tasks.

Single-digit multiplication

For reasons that will become apparent, I will distinguish three subsets of single-digit multiplication problems: (1) 0's problems (i.e., 0×0 through 0×9, and 1×0 through 9×0); (2) 1's problems (i.e., 1×1 through 1×9, 2×1 through 9×1); and (3) 2–9's problems (i.e., 2×2 through 9×9).

Arithmetic facts

Considering first the 2–9's problems, 7 of the 10 patients we have tested showed clear impairment on these problems, and for all 7 patients the impairment was non-uniform across problems. That is, for each patient the error rate was much higher for some problems than for others. This phenomenon is illustrated in Table 7, which presents the error rates for individual multiplication problems for patient PS. The table shows, for example, that PS's error rate was greater than

Table 7. *Percentage of errors for patient PS on single-digit multiplication problems*

First Operand	Second Operand 0	1	2	3	4	5	6	7	8	9
0	0	44	44	48	39	39	48	39	39	35
1	39	13	0	4	0	0	0	4	0	0
2	39	0	0	0	0	0	9	4	0	4
3	39	0	0	30	4	0	4	0	13	9
4	39	0	0	4	52	4	9	4	17	74
5	44	4	0	0	0	4	9	17	26	30
6	48	0	0	4	4	26	0	30	13	9
7	39	4	0	0	13	9	52	78	52	83
8	39	0	0	9	17	22	9	26	61	0
9	44	0	0	17	83	22	26	91	0	100

80% for 9×4, 9×7, and 9×9, but less than 30% for 9×2, 9×3, 9×5, 9×6, and 9×8.[8]

The non-uniformity of impairment has implications for models of arithmetic fact retrieval. In particular, this phenomenon poses difficulties for models assuming extensive overlap among facts in elements of the internal representations, but may readily be accommodated by models postulating a largely distinct representation for each fact (McCloskey, Harley, & Sokol, 1991).

Table-search models. Table-search models hold that arithmetic facts are stored in a table-like structure, as illustrated in Figure 4 (e.g., Ashcraft & Battaglia, 1978; Widaman, Geary, Cormier, & Little, 1989). In a typical model of this sort (e.g., Ashcraft & Battaglia, 1978) each row and column of the internal table is headed by a node corresponding to the appropriate operand, and the answer to a problem is stored at the intersection of the appropriate row and column. When a problem is presented, the corresponding row and column operand nodes are activated. Activation then spreads across the row and down the column via associative links between adjacent nodes, until an intersection occurs at the location where the correct answer is stored, as illustrated in Figure 4 for the problem 8×9.

In table-search models most representational elements and retrieval operations are shared across several stored facts. For example, the link from the 24 node to the 32 node in the 8's row of Figure 4 must be traversed in retrieving the answer

[8]Although impairment was non-uniform across problems for each patient, severely impaired and relatively intact problems were not scattered randomly over the set of 2–9's problems. In general, problems with large operands (e.g., 9×7) showed greater impairment than problems with small operands (e.g., 2×3). However, for all of the patients there were notable exceptions to this problem size effect. For example, PS made no errors on 9×8 or 8×9, but presented with a 52% error rate on 4×4. See McCloskey et al. (1991) for further discussion of problem size effects and their implications.

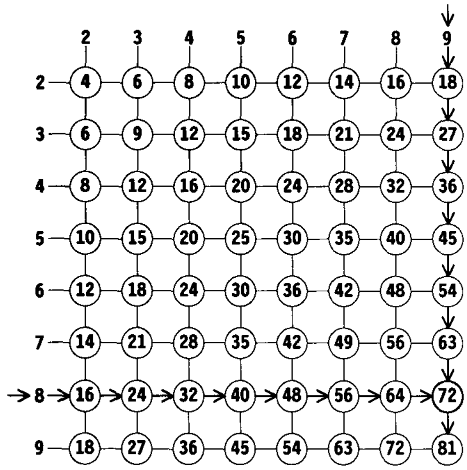


Figure 4. *Schematic depiction of a table-like representation for multiplication facts, and a table-search retrieval process.*

to 8×4, 8×5, 8×6, 8×7, 8×8, and 8×9. This feature of the models creates problems in attempting to interpret irregular patterns of impairment.

Table-search models would presumably interpret arithmetic fact retrieval deficits by assuming that brain damage disrupted the table search process, perhaps by destroying or weakening associative links between nodes in the table. For example, failure to retrieve the answer to 8×6 might occur when one or more of the links in the 8's row or 6's column could not be traversed. However, damage that would result in the patterns of non-uniform impairment observed for our patients cannot readily be specified within the table-search framework. For example, three patients (including PS) had considerably higher error rates for 8×8 than for 8×9 or 9×8. The impairment on 8×8 suggests that the search across the 8's row and/or the search down the 8's column were substantially disrupted. How, then, were the patients consistently able to retrieve the answers to 8×9 and 9×8? If the search across the 8's row were disrupted, then performance should have been impaired on 8×9; and if the search down the 8's column were disrupted, then 9×8 should have shown impairment.[9]

[9]To interpret departures from table-search predictions in the reaction time data from normal subjects, Miller et al. (1984) proposed a distinction between locating the appropriate cell in the table (through the table-search process), and accessing the answer stored at that cell. Given this distinction, irregular impairment across problems (or indeed any conceivable pattern) could be interpreted simply by assuming that the table-search process was intact, but that access to stored answers was disrupted for problems showing impairment. For example, PS's performance on 8×9, 9×8, and 8×8 could be interpreted by assuming that whereas the processes of searching along the 8's row and the 8's column were intact (allowing good performance for 8×9 and 9×8), performance on 8×8 was impaired due to difficulty in accessing the answer stored in the [8,8] cell of the matrix. However, as Siegler (1988) has pointed out, the postulation of an access process independent of the table-search process renders vacuous the assumption that retrieval involves a search through a table-like structure, because "any aspect of performance not attributable to the hypothesized [table-like] organization can be attributed to differences in accessibility" (Siegler, 1988, p. 272).

Models with distinct fact representations. Whereas non-uniform impairment across problems cannot readily be accommodated by table-search models, this phenomenon is consistent with models holding that each fact has a representation largely or entirely distinct from the representations for other facts (e.g., Ashcraft, 1987; Campbell & Graham, 1985; Siegler, 1988; Siegler & Shrager, 1984). Figure 5 illustrates Ashcraft's (1987) *network retrieval model.* Like the table-search models, the network retrieval model assumes that nodes representing problem operands are connected in a memory network to nodes representing answers. However, whereas table-search models assume that operand nodes are connected only indirectly to most answer nodes (see Figure 4), the network retrieval model assumes that each operand node is connected directly to the answer node for each problem involving that operand (see Figure 5). For example, the first-operand 6 node is connected directly to the answer nodes for the problems 6×2, 6×3, 6×4, and so forth. Thus, in contrast to table-search models, Ashcraft's (1987) network retrieval model assumes that each problem has its own operand–answer associations.

When a problem is presented, the corresponding operand nodes are activated, and activation spreads to the associated answer nodes. The most highly activated

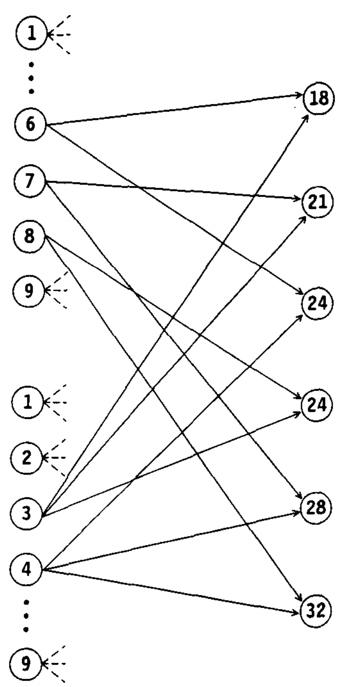


Figure 5. *A representative portion of the arithmetic fact network postulated by Ashcraft's (1987) network retrieval model. The nodes in the upper left of the figure represent the first operand in a problem, and the nodes in the lower left represent the second operand, and the nodes on the right represent answers.*

answer node is chosen as the response, if this node's activation exceeds a threshold level. Thus, presentation of 6×3 activates the first-operand 6 node and the second-operand 3 node, and activation spreads from these nodes to the answer nodes for all $6 \times N$ and $N \times 3$ problems. However, the node representing the correct answer 18 will be most highly activated, because it receives activation from both of the activated operand nodes.

Within the context of the network retrieval model or related models (e.g., Campbell & Graham, 1985; Siegler & Shrager, 1984) deficits in arithmetic fact retrieval might be interpreted by assuming that brain damage may weaken associations in the memory network. Suppose in particular that operand–answer associations are weakened in a random scattershot fashion, such that some associations are weakened to a greater extent than others. Because each problem has its own operand–answer associations, weakening of associations for a problem should impair performance only on that problem. As a consequence, random weakening of associations should lead to irregular patterns of impairment. For example, the finding that PS was severely impaired on 8×8 yet performed without error on 8×9 may be interpreted by assuming that one or more of the operand–answer associations for the problem 8×8 were disrupted, but that the associations for 8×9 were left largely intact.

Arithmetic rules

For 2–9's multiplication problems impairment was non-uniform across problems, suggesting that individual fact representations (e.g., $6 \times 8 = 48$, $6 \times 9 = 54$) underlie performance on these problems. However, the results for the single-digit 0's problems suggest a different conclusion for these problems.

For the problem 0×0, none of the patients made any errors. Hence, I will focus on the 18 problems involving 0 and a non-zero operand (i.e., 0×1 through 0×9, and 1×0 through 9×0). For 7 of the 9 patients tested extensively on single-digit 0's problems, performance was uniform across these 18 problems. (The two exceptions will be discussed below.) Three patients showed uniformly low error rates, whereas four showed uniformly high error rates. Three of these latter four patients were in fact 100% incorrect on all 18 problems. For all of the patients showing impairment on 0's problems, errors consistently took the form $N \times 0 = N$ (e.g., $0 \times 6 = 6$, $3 \times 0 = 3$).

These results suggest that 0's problems are solved by reference to a general rule (i.e., 0 times any number is 0). On this account, performance was uniform across 0's problems because a single rule mediates performance on all of the problems. (See McCloskey et al., 1991, for more detailed discussion.)

Uniform spontaneous improvement. An interesting phenomenon exhibited by patient PS offers additional support for the zero-rule hypothesis. PS was tested on

23 blocks of single-digit multiplication problems. In blocks 1–9 she erred on 98% of the 0's problems; in blocks 10–23, however, she was 95% *correct*. The performance shift was quite abrupt: PS was incorrect on all 0's problems in block 9, and correct on all of these problems in block 10. This pattern of uniform impairment followed by sudden uniform recovery may be interpreted as follows: In blocks 1–9 PS was unable to access the zero rule, and consequently showed impairment on all 0's problems. However, she regained access to the rule between blocks 9 and 10, with the result that all problems showed improvement. (See Sokol et al., 1991, for discussion of possible reasons for the sudden recovery of the rule.)

Although it has been widely assumed that 0's multiplication problems are solved by reference to a rule (e.g., Ashcraft, 1983, 1987; Ashcraft, Fierman, & Bartolotta, 1984; Baroody, 1983, 1984; Campbell & Graham, 1985; Miller et al., 1984; Parkman, 1972; Stazyk, Ashcraft, & Hamann, 1982), studies of normal arithmetic fact retrieval have not generated clear support for this assumption. In contrast, the results from brain-damaged patients provide strong evidence in favor of the zero-rule hypothesis.

One zero-rule or two? As noted earlier, two patients did not show uniform performance across the 0's problems. For these patients (FW and JB) performance was uniform across the $0 \times N$ problems (e.g., 0×3), and across the $N \times 0$ problems (e.g., 3×0), but differed between these two subsets of problems. FW presented with a uniformly high error rate on $N \times 0$ problems, and a uniformly low error rate on $0 \times N$ problems. For patient JB the opposite pattern was observed: a high error rate for each $0 \times N$ problem, and a lower error rate for each $N \times 0$ problem. This double dissociation suggests that for at least some individuals there may be two separate multiplication-by-zero rules, one applying to $0 \times N$ problems, and the other applying to $N \times 0$ problems.

Other arithmetic rules? Given the results for the 0's multiplication problems, one may ask whether other subsets of arithmetic problems may also be solved by rule. Some evidence from our patients suggests rule-based solution for several problem subsets, including 1's multiplication problems and 0's addition problems (McCloskey et al., 1991). However, the evidence is not as strong as for the multiplication-by-zero rule.

Multi-digit multiplication

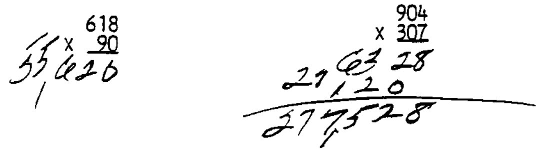
For the most part, fact-retrieval impairments evident in single-digit multiplication were also apparent on multi-digit problems. That is, the patients made the same types of fact retrieval errors in the multi-digit task as in the single-digit task. For

0's, however, dramatic dissociations were apparent between single- and multi-digit multiplication tasks.

All six patients showing impairment on single-digit 0's problems showed excellent performance in processing 0's in multi-digit multiplication problems. For example, patient GE was 0% correct on single-digit 0's problems, but 93% correct in processing 0's in multi-digit problems (Sokol et al., 1991). Figure 6 presents two examples of GE's performance on 0's in multi-digit problems.

Special-case procedures. The 0's dissociations may be interpreted in terms of *special-case procedures* for processing 0's in multi-digit problems. By special-case procedure I mean an alternate "path" in the general multi-digit multiplication algorithm, the execution of which bypasses the solving of individual 0's problems. Consider the examples of patient GE's performance in Figure 6. For the problem in Figure 6A, a person applying the general multiplication procedure would first multiply 0×8, then 0×1, then 0×6 (by retrieving the zero-rule), in each case writing the product 0 in the appropriate column. GE, in contrast, wrote a single 0 below the rightmost column of the problem, stating that ". . . this zero stands for all of the top line." He then proceeded to the next digit in the bottom number (i.e., 9), performing three fact retrievals (9×8, 9×1, 9×6), and writing answers on the same line as the 0. Thus, GE employed a special-case procedure that bypassed three individual multiplication-by-0 operations (0×8, 0×1, 0×6). GE applied this procedure to 0's in any position within the bottom operand, as illustrated in Figure 6B. Upon encountering the 0 in 307, he simply wrote a 0 (stating "zero covers everything up there"), and proceeded to the next digit in the bottom number.

Figure 6B also illustrates a second special-case procedure. Applying the general multiplication procedure to this problem would involve first multiplying 7 by 4, writing the ones digit of the answer (8) and carrying the tens digit (2). Then, 7 would be multiplied by 0, the carry of 2 would be added to the obtained answer of 0, and the resulting 2 would be written. Thus, the 8 would be written first, and the 2 would be written second. However, after GE retrieved 28 as the product of 7 and 4, he wrote "2" and then "8" (from left to right), stating that he could put



A B

Figure 6. *Examples of GE's performance on multi-digit multiplication problems involving 0's, illustrating special-case procedures.*

the whole answer down because the next digit in the top number was 0. He then proceeded to multiply 7 by 9. Similarly, in multiplying 904 by 3, he multiplied 3 by 4, writing "1" and then "2," and then multiplied 3 by 9.

On the special-case procedures account, the normal calculation system includes both the zero-rule $N \times 0 = 0$ (which is used in solving single-digit 0's problems and could be used to solve 0's problems embedded in multi-digit problems), and special-case procedures that bypass the solving of individual 0's problems in multi-digit problems. Given this assumption, 0's dissociations of the sort observed in our patients would be expected to occur in situations where the zero-rule was inaccessible (leading to errors on single-digit problems), but the special-case procedures were intact.

It is interesting to note that most of the patients presenting with the dissociation between 0's in single- and multi-digit problems showed little if any awareness of the discrepancy between their single- and multi-digit 0's responses. In the same sessions in which they consistently applied special-case procedures based upon the fact that anything times 0 is 0, they also made, with no apparent discomfort, $N \times 0 = N$ responses to single-digit 0's problems. This result seems to suggest that the special-case procedures were applied without awareness of their conceptual basis.

FUTURE DIRECTIONS

The issues I have discussed in this article will no doubt continue to occupy researchers studying normal and impaired numerical processing for many years to come. However, several additional issues are also worthy of attention.

Relation of numerical and non-numerical processing mechanisms

One issue that has not been adequately addressed in recent work concerns the relationship between mechanisms for numerical and non-numerical processing. A central question in this realm concerns whether numeral processing mechanisms are incorporated within, or are separate from, the cognitive language-processing system. Presumably, lexical processing of verbal numerals (i.e., comprehension and production of individual number words) implicates general lexical processing mechanisms. For example, in production of spoken verbal numerals phonological number-word representations are presumably retrieved from a general phonological output lexicon (although the number words may comprise a functional class within that lexicon). In the case of syntactic processing of verbal numerals, and processing of arabic numerals in general, the situation is perhaps less clear. The relationship between numerical and language processing mechanisms has been widely discussed in the dyscalculia literature (e.g., Benson & Denckla, 1969; Collignon, Leclercq, & Mahy, 1977; Hecaen, Angelergues, & Houillier, 1961), but no clear conclusions have emerged.

Another important issue concerns the role of other general cognitive capacities (e.g., working memory, spatial processing abilities) in arithmetic and other numerical processing. For example, it has often been assumed that spatial processing deficits underlie many forms of calculation disturbance (e.g., Hecaen et al., 1961; Hartje, 1987; Luria, 1966). However, the nature of the spatial processing mechanisms presumed to be implicated in calculation, and the specific forms of calculation impairment expected to result from their disruption, have not been clearly specified. (For further discussion, see McCloskey et al., 1985.)

Neural instantiation of cognitive numerical processing mechanisms

Like relationships between numerical and non-numerical processing mechanisms, relationships between forms of dyscalculia and loci of brain lesions have been extensively discussed (see Kahn & Whitaker, 1991, for a recent review). Again, however, no clear conclusions have emerged. Although several researchers have put forth hypotheses concerning lesion-deficit relationships, these hypotheses are rather non-specific, and not in close agreement with one another (e.g., Boller & Grafman, 1983; Cohn, 1961; Collignon et al., 1977; Levin & Spiers, 1985; Spiers, 1987).

The reasons for this state of affairs are not entirely clear. One possibility is that cognitive mechanisms implicated in numerical processing are not precisely localized in the brain. However, another possibility is that the failure to establish clear deficit–lesion relationships reflects the fact that most attempts to establish such relationships have relied upon rather vague and general descriptions of patients' deficits, and not the sorts of detailed, theoretically-motivated characterizations I have attempted to illustrate in this article.

Developmental dyscalculia

Dyscalculia occurs not only as an acquired disorder, but also as a developmental deficit (e.g., Cohn, 1968, 1971; Guttman, 1937; Kosc, 1974; Slade & Russell, 1971; Strang & Rourke, 1985). Systematic analyses of cases of developmental dyscalculia in light of what has been learned about normal numerical processing and acquired dyscalculia could perhaps shed light on the nature of the developmental deficits, as well as suggesting strategies for treatment. Further, the study of developmental dyscalculia, like the study of acquired dyscalculia, may contribute to understanding of normal numerical processing. Some preliminary steps in this direction have been taken (e.g., Temple, 1989, 1991), but more concerted effort is needed.

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