Title:

A novel approach to subtypes of developmental dyscalculia

Running head:

Multidimensional dyscalculia subtypes

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Abstract

Developmental dyscalculia (DD) is an heterogenous neurodevelopmental learning disability that manifests as persistent difficulties in learning mathematics. DD can occur in isolation but is often diagnosed as a co-occurring difficulty in children with language-based learning disabilities. Basic cognitive and neuroimaging findings suggest different subtypes of dyscalculia exist. However, a comprehensive theoretical framework that provides accepted terminology and clinical criteria to design appropriate interventions is still lacking.

We developed a comprehensive battery of cognitive tests, the UCSF Dyscalculia Subtyping Battery (DSB), aiming at identifying deficits in four distinct mathematical domains: number processing, arithmetical procedures, arithmetic facts retrieval, and geometrical abilities.

The mathematical abilities of a cohort of 75 children aged 7 to 16, referred to the UCSF Dyslexia Center for a language-based neurodevelopmental disorder, were initially evaluated using a behavioral neurology approach. A team of professional clinicians classified children with difficulties in mathematics in four groups, depending on their parents' and teachers' reported symptoms and clinical history, in one of the following domains: number processing, arithmetical procedures, arithmetic facts retrieval and geometrical abilities. The 75 children and 18 typically developing control children were then evaluated with the DSB to identify which subtests of the battery better represented each group. We describe the detailed profiles of four cases, each of them representative of deficits in one of the four domains, and report the pattern of impairment in the overall cohort.

Our results show that a neuroscience-based DD evaluation battery enables identification of subtypes acknowledging the multidimensional nature of the disorder. If corroborated in large samples, these findings can pave the way for novel diagnostic approaches, consistent subtype classification, and ultimately personalized interventions.

Keywords: developmental dyscalculia, number processing, arithmetical procedures, arithmetic facts retrieval and geometrical abilities

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Competing interests

The authors declare no competing interests.

1. Introduction

Despite the essential role of numerical skills in our daily life, and the recent increased awareness to mathematics learning disabilities (Gersten et al., 2007), we lack a comprehensive understanding of clinical, cognitive and neurobiological aspects of developmental dyscalculia (DD). More specifically, definitions of DD have been used ambiguously in educational, clinical, and scientific environments (Soares et al., 2018) hampering the collaborative, multidisciplinary approach necessary to translate neuroscientific findings into educational practices.

The most common definition of DD states that it is a learning disorder of neurobiological origin characterized by difficulties in acquiring adequate mathematical skills in the context of otherwise normal intelligence and age-appropriate school education (American Psychiatric Association, 2013; Butterworth et al., 2011). The predominant hypothesis in cognitive and developmental neuroscience is that DD originates from a core deficit in understanding numerosity (Butterworth et al., 2011; Dehaene, 2011). However, more recent studies indicate that children with DD may show deficits in number processing as well as in domain-general cognitive skills (Kaufmann et al., 2013; Träff et al., 2017), showing that there is no evidence for isolated core deficits in mathematical learning disabilities (Mammarella et al., 2021). Overall, the heterogeneity of DD has been largely overlooked (Geary, 2011; Price & Ansari, 2013; Träff et al., 2017). Inconsistent definitions hinder efforts to dissociate primary DD, characterized by a relatively selective deficit in numerical processing from secondary DD, in which mathematical impairments may be related to other mathematical skills, such as arithmetic procedures or facts retrieval, and may co-occur with other learning challenges, such as linguistic or attentional problems (Kaufmann et al., 2013). Furthermore, certain mathematical skills need to be taught and thus environmental factors such as lack of appropriate teaching, can cause mathematical difficulties similar to DD (Price & Ansari, 2013; Rubinsten & Henik, 2009). As a result, there are unanswered challenges in differential diagnosis with respect to other learning disorders, such as dyslexia (De Clercq-Quaegebeur et al., 2018; Guarini et al., 2021; Powell et al., 2020; Simmons et al., 2008) or ADHD (Capano et al., 2008; Peterson et al., 2017), as well as environmental factors (cultural, educational, economical, etc).

The past three decades of cognitive neuroscience research showed that DD, regardless of whether it occurs in isolation or not, is a heterogenous syndrome with different cognitive and neural mechanisms (e.g., number processing, working memory, language comprehension) that are necessary to perform specific mathematical tasks. These various processes can be difficult to tease apart using traditional evaluations thus leading to diagnostic imprecision (Geary et al., 2007; Kaufmann et al., 2013; Poletti et al., 2018). Consequently, we lack effective diagnostic tools, specific interventions and classroom didactical strategies targeting specific neuro-cognitive deficits (Von Aster & Shalev, 2007).

Therefore, there is a need for an assessment able to identify primary DD, specific to numerical processing, and also distinguish it from other mathematical deficits, that are detected in clinical evaluation (e.g., deficits in calculation, difficulties in arithmetic facts retrieval, visuospatial issues, etc.).

Converging evidence from several research groups (Butterworth, 1999; Dehaene, 1997; Geary & Hoard, 2001; McCloskey, 1992; McCloskey et al., 1985; Rubinsten & Henik, 2009; Temple, 1989; Von Aster & Shalev, 2007) highlights the key role of four basic cognitive and neural domains in the acquisition of mathematical skills: 1. number processing; 2. arithmetical procedures; 3. arithmetic facts retrieval; and 4. geometrical abilities. These domains differ in their cognitive and neural correlates, thus can be differentially impacted by neurological processes and environmental factors, and thus likely leading to corresponding, identifiable subtypes of DD.

1. Number processing. The development of the number concept requires an inherited basic *number sense* (Cohen & Dehaene, 1995) or *number module* (Butterworth, 1999) and the

acquisition of its exact numerical and linguistic representations. Converging evidence suggests that humans are born with the capacity of representing numbers as continuous quantities along a spatially organized *mental number line* (de Hevia et al., 2014; Dehaene, 1992; Izard et al., 2009; Piazza et al., 2004). The foundation of the number concept has also been linked with two nonverbal systems: subitizing (i.e., the ability to automatically identify the cardinality of small sets of objects; Piazza, 2010) and the approximate number system (ANS, i.e. the ability to estimate the numerical magnitude of big sets of objects; Dehaene, 1997). These systems are present from infancy (Carey, 2002; Feigenson et al., 2004; Hyde, 2011; Izard et al., 2009; Xu, 2003) and are shared with a wide variety of other animals (Brannon & Merritt, 2011; Feigenson et al., 2004). While it is subject of debate whether subitizing abilities are linked to counting skills and other non-numerical capacities such as attention and working memory (Anobile et al., 2019; Arrighi et al., 2011; Piazza et al., 2011), the ANS is considered a core component of numerical cognition development (Fazio et al., 2014; Izard et al., 2009; Piazza, 2010; Schneider et al., 2017). A series of studies have found that ANS acuity predicts later mathematical achievement (Feigenson et al., 2013; Mazzocco et al., 2011), and it is frequently found to be impaired in children with mathematics learning disorder (Piazza, 2010; Wilson & Dehaene, 2007).

The brain network related to number processing mainly involves the lateral parietal lobe, but also extends to inferior frontal gyri, the insula and subcortical structures (Menon, 2015). Which such an extended network of brain regions, it is very likely that problems in mathematical reasoning might arise from different underlying computations and therefore manifest in different behavioral phenotypes.

Deficits in number processing, characterizing primary DD, manifests with several symptoms in learning mathematics, including manipulating and transcoding numbers (Moura et al., 2015). A lack of understanding the concept of numerosity prevents normal development of number representation (Geary, 1993), number production and comprehension (McCloskey, 1992; McCloskey et al., 1985; Temple, 1989). Children with deficits in number processing manifest impairments in transcoding numbers across representational formats (e.g from pictorial or words to Arabic numeral), as well as in comparing and ordering quantities (Attout & Majerus, 2015; Mussolin et al., 2010). In counting, process deficits are mainly detected in counting backward, counting on by twos or by threes and filling out sequences (Landerl et al., 2004).

Clinical assessment of deficits in number processing should thus include subitizing, ANS and counting principles, comparisons, ordering, and transcoding between number digits and number words.

2. Arithmetical procedures. Calculation skill is the ability of adding, subtracting, multiplying, or dividing using mental and written strategies, usually without the need of counting. This skill is necessary to correctly provide the result of a mathematical task and it is important to speed up the solving process. The activation of appropriate numerical representations is one of the prerequisites for the correct execution of any calculation (Ansari, 2012; Piazza et al., 2010). However, calculation deficits may be observed even if numerical skills are not compromised (Ganor-Stern, 2017; Jordan & Montani, 1997; Russell & Ginsburg, 1984). These deficits may emerge as relatively isolated struggles in solving arithmetic problems (Geary, 1993; Pesenti et al., 1994; Temple, 1991).

The brain network underlying calculations appears to involve a large set of interconnected fronto-parietal cortical and subcortical regions, as well as medio-temporal structures such as the hippocampus (De Smedt et al., 2010; De Smedt & Boets, 2010; Dehaene et al., 2003; Grabner et al., 2009; Peters & De Smedt, 2018).

Finger counting is frequently observed in children who exhibit deficits in arithmetical procedure (Attout & Majerus, 2015), possibly to decrease the burden on working memory while attempting to process the calculation (Crollen, 2011; Vandervert, 2017) or overreliance on immature calculation strategies. Deficits in arithmetical procedures also appear when children fail to acquire efficient calculation strategies (Rapin, 2016; von Aster, 2000), which are known to follow specific developmental trajectories (Robinson et al., 2006). For instance, a child might fail to move from counting with external aids (e.g., fingers) towards more complex mental strategies such as counting-on (i.e., to perform 2+7=9 the larger addend 7 is identified and 2 is added counting on eight and nine) (Fuson, 1982; Geary et al., 1992).

The clinical assessment of calculation skills should cover mental calculations, in particular additions and subtractions, as well as written calculations involving all four operations.

3. Arithmetic fact retrieval. Arithmetic facts are simple operations that children are expected to memorize and store in long-term memory (Carpenter & Moser, 1984; De Visscher &

Noël, 2013; Jordan & Montani, 1997). Memorization of arithmetic facts is necessary to perform quick calculations that are the building block of mathematical competences. For instance, children cannot learn to perform written multiplication and divisions without knowing multiplication tables. Deficits in learning arithmetic facts are sometimes found as isolated difficulties in mathematics (Geary, 1993; Pesenti et al., 1994; Temple, 1991).

Arithmetic facts retrieval and calculation are associated with overlapping yet and distinct brain areas (Dehaene & Cohen, 1997; Semenza et al., 1997). While calculations have been linked to the above described fronto-parietal network, learning and retrieval of arithmetic facts appears to be supported by hippocampal, para-hippocampal, and retrosplenial structures (Bloechle et al., 2016) with critical contributions from the angular gyrus (Grabner et al., 2009).

Children who exhibit deficits in arithmetic facts usually struggle in memorizing simple addictions and subtractions (e.g., 2+2=4, 3-1=2) or in retrieving multiplication tables (e.g., the tables of 6, 7 and 8). However, if number processing and arithmetical procedures skills are preserved, they might be able to reconstruct the operation's result using counting and mental strategies.

Clinical assessment of arithmetic facts retrieval should include the evaluation of arithmetic facts knowledge. Testing should cover multiplication tables at appropriate age and educational level.

4. Geometrical abilities. Difficulties in mathematics might still be observed in children with intact numerical skills, calculation ability, and memory functions to recall arithmetic facts. In this case, deficits often reside in the non-linguistic aspects of mathematical information (Rourke, 1993) such as visuo-perceptual, visuo-spatial, and visuo-constructional skills. These nonverbal abilities have been shown to predict mathematical performance (Shea et al., 2001), particularly geometry acquisition (Battista, 1990). Geometrical knowledge is thought to reside on two core knowledge systems: an innate sensitive skill to quantitatively process visual properties (e.g., length, angle, and shape), and a later life developed ability targeted to process specific geometrical concepts (e.g., parallel vs. perpendicular lines) (Izard & Spelke, 2009).

Visuoperceptual, visuospatial, and visuoconstructional abilities have been localized in occipito-temporal (Epstein & Kanwisher, 1998; Kanwisher et al., 1997), temporo-parietal (Kravitz et al., 2011) and parietal (Galletti et al., 1997) cortices respectively, with a right hemisphere lateralization.

Deficits in geometrical abilities can manifest with visuoperceptual faults in processing symbols, arranging numbers for written calculations, interpreting graphs and figures, and recognizing salient visual features of objects (Geary, 1993). Symptoms might be also present in processing visuospatial information such as spatial reasoning about orientations, directions, and distances, as well as visuoconstructional information necessary to transform three-dimensional objects (Kinach, 2012; Simic et al., 2013; Stern et al., 2015).

Clinical assessment of geometrical abilities should include tasks requiring (1) processing distances and directions, targeting visuospatial ability (2) matching shapes presented in different orientations or decomposed to demonstrate visuoperceptual ability, and (3) mental rotations of 2D and 3D objects to assess visuoconstructional functions (Bruce & Hawes, 2015). Stimulus in 3D and 2D are processed differently in the brain (McGraw, 2004), e.g., 3D imposes a higher load on perceptual system, but it is memorized better than 2D (Nejati, 2021).

In addition to these different domains affecting mathematical cognition, the relation between linguistic and mathematical aspects of cognition is still a matter of debate. Of particular clinical and educational relevance is the overlap between dyslexia and dyscalculia. The co-occurance of dyslexia and dyscalculia is reported to be surprisingly high with around 40% of children with reading difficulties also having difficulties in learning math, presumambly not only specific to number processing (Butterworth, 1999; Dirks et al., 2008; Wilson et al., 2015). Difficulties in retrieving arithmetic facts from long term memory (Träff & Passolunghi, 2015) are common in dyslexia (De Smedt et al., 2010). Impairments in specific cognitive domain such as verbal working memory, visuospatial working memory and lexical naming speed could also explain the co-occurnce of defects in mathematical cognition and reading challenges (Bull et al., 2008; Wilson et al., 2015).

Comprehensive studies that have systematically investigated different components of mathematical skills in the same cohort of children are lacking. To address this knowledge gap, we designed and tested a novel battery of tests, the UCSF Dyscalculia Subtyping Battery (DSB) specifically designed to comprehensively assess the four mathematical domains described above: 1. number processing; 2. arithmetical procedures; 3. arithmetic facts retrieval; and 4. geometrical abilities. Here we describe the battery, the neurocognitive profiles of four exemplary cases with different kind of mathematical difficulties and report results obtained in a large cohort of children (n=75) evaluated at the UCSF Dyslexia Center for a variety of cognitive developmental difficulties mainly related to reading and 18 typically developing control children, focusing on demonstrating the diagnostic utility and feasibility of the DSB battery.

2. Materials and Methods

2.1 Participants

Participants were recruited through the UCSF Dyslexia Center (UCSF-DC), a multidisciplinary research center dedicated to the study of dyslexia and related neurodevelopmental cognitive disorders. At the UCSF-DC participants referred for a concern of learning disorder underwent an extensive research evaluation by a team of clinicians, including neurologists, neuropsychologists, genetic counselors, speech and language pathologists, psychiatrists and educational specialists. The team provides an overall diagnostic impression based on clinical history (first symptoms and most severe impairments from parents and teachers), family history (similarity between siblings and/or parents), standard neuropsychological and academic testing and questionnaire responses (items indicating clinical significance). Participants were excluded from the study if they showed borderline or impaired general cognitive scores, known history of severe perinatal events such as strokes, an acquired brain injury, genetic, neurological, or psychiatric disorder associated with seizures, impaired sensory processing or communication. Inclusion criteria included fluent in English, and have an age between 7 and 16 years. Typically developing control participants had no subjective concerns about academic achievement, no prior diagnoses of neurodevelopmental disorders, age ranging between 7 and 16 years, and fluency in English.

The final study cohort included 93 children, 18 typically developing children (female = 7, mean age = 10.40 (± 1.66), 94% right-handed) and 75 were diagnosed by the team of UCSF-DC as having a focal neurodevelopmental disorder causing a learning disorder (female = 27, mean age = 11.78 (± 2.05), 92% right-handed). See Table 1 for detailed demographics. The clinical group was composed by a total of 50 chidlren with dyslexia only, 7 children with ADHD and 18 children with both dyslexia and ADHD. The diagnosis was performed by a multidisciplinary team and using the IDA criteria for dyslexia and the DSM-V criteria for ADHD.

Most children of the clinical group (57/75, 76%) attended independent schools for children with learning differences; teachers were able to provide detailes descriptions of their academic challenges.

Guardians of the participants provided informed written consent and participants provided assent. The study was approved by the University of California San Francisco (UCSF) Institutional Review Board and complied with the declaration of Helsinki.

2.2 *Neuropsychological and Academic Assessment*

Neuropsychological and academic testing were administered or supervised by a licensed neuropsychologist. The tested covered screening of nonverbal reasoning, vocabulary, processing speed, attention and working memory, verbal and visual recall, visuospatial and visuo-construction abilities, and executive functions (Beery, 1997; Dean et al., 1994; Korkman et al., 2007; Llorente et al., 2003; Martin & Brownell, 2010; Meyers & Meyers, 1995; Wechsler, 2014; see Supplementary Table 2 for a full list of tests). Academic testing was done using the Woodcock-Johnson IV (WJ-IV) (Schrank et al., 2014). In addition to some of the untimed reading measures in the WJ-IV, participants were also administered the Test of One-Word Reading Efficiency, version 2 (TOWRE-2) (Torgesen et al., 2012).

2.3 *Study Procedure*

Clinical classification. As part of the UCSF-DC diagnostic process, the team of clinicians considered each child's clinical history, teacher interviews, cognitive and academic evaluations, and determined whether they showed difficulties in mathematical cognition, and if so, which aspect was most involved.

From this clinical assessment, 50 of the 75 participants with diagnosis of a learning disorder were classified as having difficulties in some aspect of mathematics (66.6%). Of these 50 children, 10/50 (20%) were judged to have an overall impairment in mathematics with difficulties in understanding basic concepts such as numbers, magnitudes, measures. These children were described as failing to correctly process numbers, e.g. teachers usually described these children as having trouble in connecting the number to the corresponding magnitude, parents recalled times during early learning when their child struggled in counting, or in understanding differences in quantities of objects. The team classified these participants as having troubles in number processing. Most of the children struggling in mathematics (32/50, 64%) appeared to understand numbers but still have difficulties in calculation activities, 16 mostly when performing mental or written calculations, and 16 impairments in memorizing multiplication tables and math facts. The team classified these participants as having deficits in arithmetical procedures and arithmetic fact retrieval respectively. Finally, 8 of the 50 children (16%) struggling in mathematics were described as having trouble in mathematical activities involving orientation, direction, distance and in processing visuospatial information (e.g. reading graphs, comparing similarities and differences in figures, etc.). These children did not have trouble with calculations but struggled to understand mathematical concepts in visual forms. These participants were classified by the team as having visuospatial troubles, identified by us as impairements in geometrical math abilities.

In summary, from the clinician assessment, 25/75 children were classified has not having trouble in mathematics (33.3%). Of the 50 children with math difficulties, 10/75 were classified with impairments in number processing, i.e. were diagnosed as having primary dyscalculia (13.3%), 16 in arithmetical procedures (21.3%), 16 in arithmetic fact retrieval (21.3%) and 8 in geometrical abilities (10.7%). The UCSF Dyscalculia Subtyping Battery (DSB) was not used for this first evaluation.

2.4 The UCSF Dyscalculia Subtyping Battery (DSB)

After this initial clinical assessment, the 75 participants and the 18 typically developed children were tested with the UCSF Dyscalculia Subtyping Battery (DSB) to assess detailed aspects of the different mathematical processing domains. The DSB is a detailed experimental battery developed to discriminate difficulties in different mathematical domains and identify mathematical strengths and weaknesses of each participant. The DSB is designed for students from the 2nd to the 8th grade, with seven different forms based on grade level. Overall, it includes 4 computer-based tasks and 15 paper-based subtests targeting number processing, arithmetical procedures, arithmetic facts retrieval and geometrical abilities. In addition, 8 subtests are included to evaluate more complex mathematical skills (e.g, simplifying expressions, solving equations and geometrical problems) and to verify adequate teaching exposure (the lack of which would prevent a diagnosis of learning difference in mathematics). Examples and details are reported in Supplementary Table 1.

Eight subtests were designed to evaluate the ability to recognize and compare magnitudes expressed via digits, words, or arrays of dots: five paper-based problems were developed, and three computer-based tasks were adopted from previous literature (Dehaene et al., 2006; Izard & Spelke, 2009; Pinheiro-Chagas et al., 2014).

Four subtests were designed to assess calculation abilities: three address mental calculation skills and the last one, written calculation abilities. The addition and subtraction problems on the calculation subtests were designed to target difficulties with arithmetic procedures. A mental multiplication subtest and the multiplication and division problems in the written calculation subtest should evaluate arithmetic facts retrieval skills. Additionally, the time to complete the calculation subtests was recorded in order to assess the fluency with which someone completes the subtests, which may help to differentiate difficulties in arithmetic procedures and arithmetic facts retrieval.

Finally, a computerized task (Izard & Spelke, 2009) and three paper-based subtests were designed to assess geometrical abilities. These included items that varied on symmetry, rotation, shape, angles, etc.

Three tasks were added to evaluate mathematical abilities for older children (e.g., simplifying expressions, solving equations, modeling) who might have compensated basic deficits but reveal difficulties in more complex problems. Seven additional tasks were designed to verify adequate teaching exposure, to confirm or exclude specific deficits, e.g. fractions. For example, a deficit in arithmetic procedure may be confirmed in solving word problems if the adequate operation is identified but the calculation is incorrectly performed.

On average, participants were tested for a total of 1 hour and fifteen minutes. Performance and time were recorded for each subtest. Each child was assessed with the battery tailored to the child's grade level. Children tested during the first three months of the scholastic year (until Christmas break) were evaluated with the battery tailored to their previous grade level to help alleviate didactical confounds. In total, 12 children were tested on the 2nd grade battery form, 6 on the 3rd grade form, 16 on the 4th grade form, 23 on the 5th grade form, 12 on the 6th grade form, 8 on the 7th grade form, and 16 on the 8th grade form.

2.5 Statistical Analysis

Group differences in DSB were evaluated to identify specific tasks that matched with the expert clinicians' assessment. Demographic, neuropsychological, and DSB measures were compared across groups (Table 1). Data were analyzed using Stata 15 (StatCorp, College Station, TX). ANOVA, independent sample student's t-tests, and chi-squared analyses were used for parametric and non-parametric data respectively, using tests for unequal variances as appropriate. All pairwise and multiple comparisons used a Bonferroni correction. All statistical analyses were reviewed and if necessary, revised by a statistician. The statistical analysis for group performance in the UCSF-DSB is presented after a general description of the performance in clinically defined groups and a presentation of four single cases representative for each group (e.g., impairement in number processing, impairement in arithmetical procedures, impairement in arithmetic facts retrieval and impairement in geometrical math abilities).

3 Results

Description of UCSF-DSB performance in clinically defined groups

The UCSF-DSB confirmed the clinicians assesment for the 18 tipically developing control children and the 25 children classified with learning disorder and not having trouble in mathematics (LD not Math). These participants performed well on the DSB subtests evaluating number processing, arithmetical procedures, arithmetic facts retrieval and geometrical abilities. Some of these participants obtained low scores only on subtests assessing teaching exposure (e.g., 8% obtained low score in the "equivalent fractions" subtest and 18% in the "percentage" subtest) or more complex mathematical skills (e.g., 30% obtained low score in the "simplifying expressions" subtest and 36% obtained low score in the "solving equations" subtest). The other 50 participants from our learning disorder cohort (50/75, 66.6%), classified from the clinicians' team as having deficits in mathematics, performed poorly on specific subtests of the DSB in accordance with their specific deficit.

Participants classified as having deficits in number processing (n = 10/75, 13.3%) showed a systemic pattern of weakness in at least three of the number subtests (from 1 to 8 in the Supplementary Table 1) or by severe deficit in comparing, ordering or transcoding numbers from a representation to another with lexical (digits are incorrectly chosen but the order of magnitude is correct, e.g., *250* instead of *215*), or syntactical (digits are correctly chosen but the order of magnitude is incorrect e.g., *200053* instead of *2,053*) mistakes. Participants with deficits in number processing obtained low scores in most of the DSB's subtests. Numbers are at the base of calculation, logical reasoning and problem solving; a deficit in numbers affects much of mathematical thought (Butterworth, 2005). Moreover, prior research that found a positive relationship between numerical and visuospatial skills (Sella et al., 2016) may help to explain the low scores obtained by these participants in the geometrical subtests.

Participants with deficits in arithmetical procedures (n = 15/75, 20%), showed a systemic pattern of weakness in the calculation subtests (9, 10, 12, 13 in the Supplementary Table 1) or in accurate but slow calculation, in particular impairments were observed in adding and more often subtracting numbers. These children performed well in number processing subtests.

Participants with deficits in arithmetic facts retrieval (n = 15, 20%) obtained low scores or accurate but slow performance on multiplication and division problems (11 and 13 in the Supplementary Table 1).

Participants with deficits in geometrical abilities (n = 7, 9.3%) obtained low scores on the computerized geometrical test (27 and 13 in the Supplementary Table 1) or they showed a pattern of weakness in the geometry subtests (22, 23, 25, 26 in the Supplementary Table 1). Visual impairments were of course excluded.

Three participants (4%) were excluded from further analyses because their deficits overlapped in arithmetic procedures and fact retrieval and a clear subgroup could not be determined.

In the next section we present the neurocognitive profiles of four exemplary cases, selected from the whole cohort of children, each of them representative of one of the four mathematical deficits (Fig. 1). For each case, we analyzed their performance on the DSB, including a description of their mistakes to distinguish the characteristics of each mathematical deficit.

3.1 Individual Case Results

The four cases presented in this paper, are in 4th grade. They were chosen because they all attended independent schools for children with learning disorder, they had the same age at the time of the assessment, and each case was representative of a specific impairment in mathematics. Strengths and the weaknesses for each child are summarized in Fig. 1.

“AM”: Number processing impairment (F, 11 years and 2 months, right-handed)

AM’s difficulties in school were first discovered when she was in kindergarten and her teachers noticed that she was far behind her schoolmates in learning and development. This led to an evaluation while she was in kindergarten where she was diagnosed with specific learning disorder in reading (dyslexia) only. However, in 3rd grade, she was assessed again, and it was noted that she had specific deficits in mathematics. Her evaluation consisted of the Wechsler Individual Achievement Test – III (WIAT – III). She obtained low scores in math operations skills, math reasoning, and in the speed of retrieval of basic facts.

At the UCSF-DC, she was classified in the dyslexia group and identified as having impairments in math.

The team’s clinicians identified relative strengths in verbal and visual memory skills and weakness in phonological processing, rapid retrieval of verbal information, and in tasks that relied on visual motor integration and spatial analysis.

From the clinician’s assessment of the UCSF-DC, she was classified as having difficulties in number processing. On the Woodcock Johnson-IV Test of Academic Achievement, Calculation subtest, *AM* obtained a raw score of 19, which translates to a percentile score of 7 (administered at age 9). This in the borderline impaired range, which suggests that she has an unspecified difficulty in mathematics. In addition to that, teachers’ and parents’ observation emphasized her rejection of mathematics. It was common for *AM* to cry and complain about math homework. She struggled in learning new mathematical concepts such as negative numbers, and fractions. The parents recalled how hard it was for her to learn to count.

On the DSB (administered at 11), *AM* showed the most difficulty in number subtests. She struggled converting numbers between digit form and written words and vice versa. *AM* made lexical (e.g., she wrote *19* for *eighty-nine* or *sixty-six* for *69*) and syntactical mistakes (e.g., she wrote (*two thousand million, two hundred thousand fifty, two hundred* for *2,250,200*). This demonstrates a weakness in recognizing the structure and the meaning of the number (Geary & Hoard, 2001; Temple, 1989). She also struggled to associate quantities with numbers. In the subitizing test, students who completed the DSB generally identify up to four dots 100% of the time and this has been found to be typical of most children (Anobile et al., 2019). *AM* correctly identified four dots 60% of the time. Additionally, she had crude estimation skills. When *AM* was presented with eight or nine dots, she would sometimes estimate around 20 or 30 dots. Similarly, in the ANS, she correctly selected 46 stimuli out of 64 in dot comparison, and 44 stimuli out of 64 in digit comparison, which is around 17% and 30% less than the average of her same aged peers performance on the DSB, respectively.

As expected, because numbers are related to other math skills, *AM* obtained low scores in all the other areas of the battery including calculations and geometrical awareness. Therefore, *AM* demonstrated what we believe is a prototypical pattern of mathematical difficulties arising from a deficit in number processing. Primarily, she displayed challenges in the number subtests and then subsequently lower scores in the other three domains as well. This profile reflects what many researchers, but not necessarily practitioners, refer to as dyscalculia.

"BN": Arithmetical procedures impairment (M, 10 years and 7 months, right-handed)

Concerns about *BN*'s learning first arose when he was around 3-years-old and his parents noticed he had challenges in differentiating numbers from letters. He struggled throughout preschool, especially in tasks that relied heavily on concentration and writing. *BN* underwent neuropsychology assessment in second grade and was diagnosed with ADHD and dyslexia. Arithmetic was one of his least favorite subjects but mathematical difficulties were not clinically diagnosed.

At the UCSF-DC, *BN* was classified in the dyslexia and ADHD group and identified as having impairments in math. The team's clinicians identified relative strengths in phonology processing, visuospatial functions and semantic fluency and weakeness in executive function, sustaining attention, and visuomotor integration skills.

From the clinicians' assessment of the UCSF-DC, this child was classified as having trouble with arithmetical procedures. On the Woodcock Johnson-IV Test of Academic Achievement, Calculation subtest, *BN* obtained a raw score of 23, which translates to a percentile score of 7. This in the borderline impaired range, which suggests that he has an unspecified difficulty in mathematics. In addition to that, *BN*'s math teacher and his parents remarked that they noticed *BN* could not perform calculations without counting on his fingers, which was a behavior most of his peers had already abandoned. He also needed a calculator to complete any mathematical assignments sent home for homework. However, his teacher emphasized that *BN* understood mathematical concepts, had a well-established sense of magnitude, and had good reasoning skills.

On the DSB, *BN* did well on number subtests (in contrast to *AM*). In the ANS, he correctly selected 51 stimuli out of 64 in dot comparison, and 63 stimuli out of 64 in digit comparison. *BN* showed the most difficulty in calculation subtests. In particular, he struggled to add and subtract numbers. He did not use mental strategies to perform these operations. He used his fingers to count, and the counting forward strategy to perform additions and the counting backward strategy to perform subtractions. For example, he counted backward from 16 (16, 15, 14…) to solve the problem 16-7=9. For his age, this is an immature approach that is inefficient for larger calculations and can lead to errors even in small calculations. For 2-digit calculations, that most children of his age do mentally (e.g., 26+14, 30+20 13+12), he arranged the numbers in columns so he could continue to use his fingers and the counting forward strategy to add them. He was able to correctly add two-digit numbers if he wrote them in columns. However, this approach did not help him on two-digit subtraction problems. He did not provide any answers for 2-digits subtractions (e.g.: 35-15, 48-28, 100-55 and 100-78). In written calculations, he made calculation errors (e.g., 18+27=44, he incorrectly added 8+7 in the second column; in 90-47=57 he incorrectly subtracted 0 from 7 instead of 7 from an assumed 10). *BN* was inconsistent but not impaired in his performance in single digit multiplication problems, which suggests a minor weakness in arithmetic facts but not a full deficit as will be described for the next student. He obtained average score in the Geometry subtests. This is congruent with previous research indicating that geometrical competences are independent of calculation abilities.

“CJ”: Arithmetic facts retrieval impairment (M, 10 years and 6 months, right-handed)

Concerns about *CJ*’s learning first arose when he was in 2nd grade and he started receiving reading support at school. In 3rd grade, he underwent neuropsychological testing and was diagnosed with specific learning disability in math and reading. He received speech therapy and intervention in reading. No specific interventions were mentioned for his specific learning disorder in mathematics.

At the UCSF-DC, *CJ* was classified in the dyslexia group and identified as having impairments in math. The team’s clinicians identified relative strengths on phonology, phonological loop and memory skills. Weakness were identified in reading, word retrieval skills and frontal/executive functioning. From the clinicians’ assessment of the UCSF-DC, he was also classified as having trouble with arithmetic facts retrieval. On the Woodcock Johnson-IV Test of

Academic Achievement, Calculation subtest, *CJ* obtained a raw score of 24, which translates to a percentile score of 12. This in the low average range, which suggests that he has an unspecified difficulty in mathematics. His parents confirmed that he never learned multiplication tables. However, his teacher observed that *CJ* could reconstruct the result of the multiplication using calculation strategies (e.g., 6x9=54 because he knew 6x10=60 and he subtracted 6 from 60).

On the DSB, *CJ* performed well on number subtests. In the ANS, he correctly selected 41 stimuli out of 64 in dot comparison, and 51 stimuli out of 64 in digit comparison. He also demonstrated an ability to use mental strategies for calculations but needed time to apply them. He showed the most difficulty in arithmetic facts subtests. He struggled to recall the multiplication tables of 6, 7, 8, and 9. This impeded his ability to perform multiplication problems because he would instead attempt to write out small addition problems that he could then add together to solve a larger multiplication problem, which strained his working memory, e.g. for 38x4, he added 30+30=60, 30+30=60 (30 added 4 times), then 60+60=120, then 8+8=16, 16+16=32, (8 added 4 times) and finally 120+32=152. This process demonstrates that *CJ* understands numbers and how to calculate through a multiplication problem but is slowed down and at risk of making an error through numerous steps. Whereas if he had memorized arithmetic facts this would have been a simple two-step operation. Similar to *BN*, he obtained an average score in the geometry subtests, another example of the independence of geometry skills from calculations and arithmetic facts.

“DJ”: Geometrical abilities impairment (M, 10 years and 5 months, right-handed)

Concerns about *DJ* first arose when he was 12 months old and throughout his early development he struggled with speech and language. When he was in 1st grade, he received an IEP and began being pulled out of the classroom for services in reading, speech, and OT. When he was around 7 years old, he received an educational evaluation based on the Wechsler Individual Achievement Test – III (WIAT – III). He received an average score in math problem solving, numerical operations, and math fluency addition and subtraction. Importantly, in the WIAT-III math problem solving subtest, he demonstrated a good understanding of mathematical calculation skills but had some difficulties in telling time. When shown a clock and asked to tell the time he did not know the answer.

At the UCSF-DC, he was classified in the dyslexia groups and identified as having impairments in math. The team’s clinicians identified relative strengths on semantics, vocabulary and visual memory skills and weakness on phonological loop (or auditory-verbal short-term memory) related tasks.

From the clinicians' assessment of the UCSF-DC, he was also classified as having trouble with geometrical abilities. On the Woodcock Johnson-IV Test of Academic Achievement, Calculation subtest, *DJ* obtained a raw score of 26, which translates to a percentile score of 21. This is in the low average range, which suggests that he has an unspecified difficulty in mathematics. His parents noticed that since his early age, he struggled in solving orientation tasks (e.g, recognizing places in a city, identifying directions) and he did not like to play with games involving visuospatial functioning (e.g., assembling and decomposing figures, puzzles, etc.). His teacher observed that he had troubles in reading graphs.

On the DSB, *DJ* performed well on most of the number subtests. However, in the approximate number system test he correctly selected 33 stimuli out of 64 in dot comparison, and 46 stimuli out of 64 in digit comparison. The dots comparison score is low in respect to the average of his same aged peers. This pattern is also in contrast to the pattern seen in *AM*'s case (low number sense) because *DJ* has substantially more difficulty with dots than digits whereas *AM* does poorly with both. This confirms that *DJ*'s geometrical competencies are impaired but not his numerical skills. Additionally, he did well on the subitizing' task (up to 5 dots), which again indicates that his numerical skills are intact. Although the subitizing task utilizes dots, the response is based on rapid assessment of a number quantity rather than a relative estimation of magnitude, which are not necessarily reliant on the same cognitive processes and may show a dissociation between number processing and geometrical abilities. Another indication that *DJ*'s difficulties lie in the visual domain is that he performed very well in calculations and arithmetic facts subtests. Results from our battery revealed that *DJ*'s main weakness in mathematics is specifically related to geometrical issues. Paradoxically, *DJ* performed well in the subtest requiring identifying a 3D model from a 2D unfolded shape. However, as mentioned above in the research section, there is a difference in how the brain processes 2D and 3D objects (McGraw, 2004; Nejati, 2021). More notably, *DJ* struggled in mentally assembling figures. When he was asked to state if a figure could be assembled using other figures, he would rotate the paper and redraw the figures in a different orientation than originally presented. *DJ* made a drawing, and he stated his answer "N" to indicate that the two trapezoid figures cannot reconstruct the hexagonal figure, which is not correct (as shown in the Supplementary Table 1: "incorrect response" of task 25). Typically, children his age mentally rotate one of the initial figures 180 degrees horizontally to match the horizontal orientation of the target figure and correctly state that these two initial figures can make the target. *DJ* drew both initial figures with a 90-degree vertical rotation and his final figure adjacent to the target but also 90 degrees rotated. He never drew a final figure in the same orientation as the target. *DJ* was unable to provide a correct answer in these cases (e.g., he also failed in selecting a triangle from a set of quadrilaterals, or an ellipse from a set of circles). In general, errors were mainly due to difficulties in recognizing similar figures, angles, rotations, and symmetries.

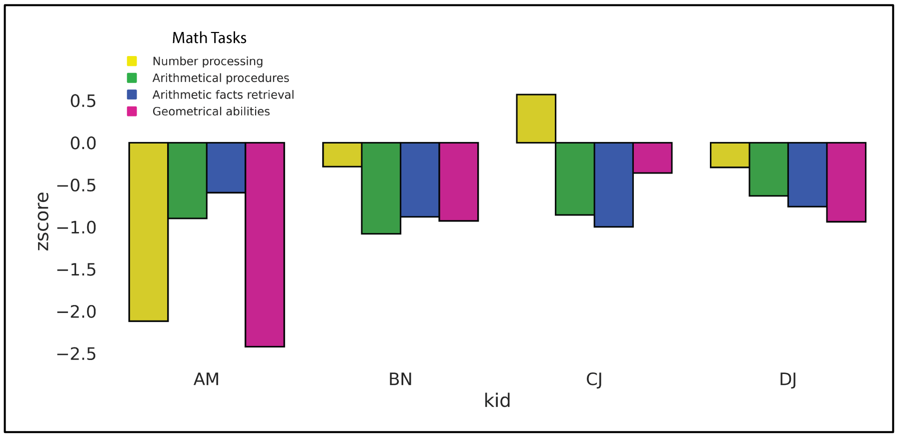

Figure 1. Exemplar cases of mathematical impairments. Standard score performance on tasks weighing on number processing (yellow), arithmetical procedures (green), arithmetic facts retrieval (blue), and geometrical abilities (purple), for the four single cases described (AM, BN, CJ, DJ). Details on the different subtests (including examples of incorrect responses) are in Supplementary Table 1.

3.2 Group Performance in DSB

There were no group differences noted on the demographic characteristics (e.g., sex, handedness, and age at testing). As expected and defined, there was a significant group difference on diagnoses. When the controls were removed from the analysis, there was no longer a significant group difference in diagnoses (p=0.288).

There are no significant differences in the distribution of subtypes of mathematical deficits across grades (p=0.83), however it should be noted that this analysis is possibly underpowered given the relatively low number of cases in some subtypes. There were significant group differences in all subtests in the DSB math battery (Table 1) except for the 2D shape reconstruction test.

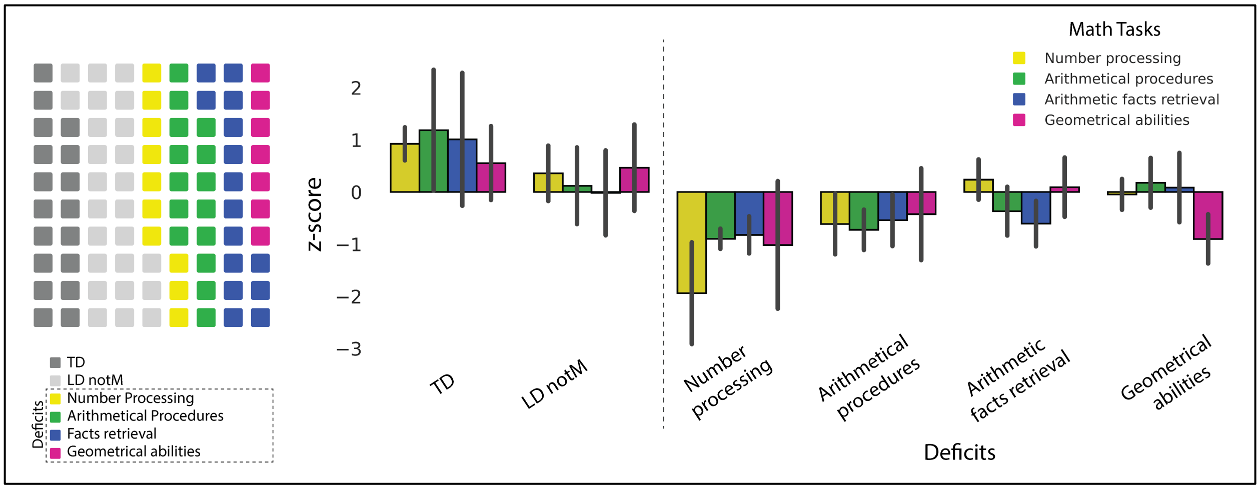

Figure 2. Group performance in DSB. (a) The distribution of each group across our sample (n=90): deficits in number (n = 10, 11%), deficits in arithmetical procedures (n = 15, 17%), deficits in arithmetic facts (n = 15, 17%), or deficits in geometry (n = 7, 8%). (b) Standard score performance on tasks weighing on number processing (yellow), arithmetical procedures (green), arithmetic fact retrieval (blue), and geometrical abilities (purple), for the six groups identified: typically developing (TD) children, children with learning disorder (LD) not affecting mathematical abilities, children with deficits in number processing, arithmetical procedures, arithmetic facts retrieval, and geometrical abilities. Details on the different subtests (including examples of incorrect responses) are in Supplementary Table 1.

Post-hoc analyses revealed that participants with a learning disorder but not struggling in math (LD_notM group in Table 1) performed similarly to the typically developed group (TD group in Table 1) in all subtests except ANS for digits. The TD group performed significantly better than all other subgroups on the ANS for digits. It is not surprising that LD_notM group performed worse than TD group in this subtest because 27 children with LD_notM have ADHD, and attention deficits may affect the ANS score. Additionally, performance on the subitizing subtest revealed a main effect of group (p=0.031), but post-hoc analysis indicated a specific group effect only for TD compared to all other groups combined (TD mode = 6 dots; all other group modes = 5; p=0.016).

<u>Children with deficits in number processing</u> (n = 10, 11%) performed significantly lower than all other groups on number subtests as well as the addition and subtraction subtests of the calculations section and the multiplication subtest of the arithmetic facts section, which suggests that deficits on number processing represent more fundamental deficits in learning mathematics. They also performed worse than TD and LD_notM subgroups on the ANS for digits.

<u>Children with deficits in arithmetical procedures</u> (n = 15, 17%), performed similarly to TD group on two of the four number subtests, accuracy but not time on the addition subtest, accuracy

and time on the multiplication subtest, and both 2D and 3D models. They had low scores, similar to children with deficits in number processing, on the sequencing subtest; written calculation and geometrical test accuracy; as well as time on additions and subtractions. Notably, the group of children showing deficits in arithmetical procedures performed significantly differently than all other groups on the subtraction subtest: the reaction time is the highest for all groups and the performance is lower than the other groups, except for the group with deficits in number processing. In general, the group showing deficits in arithmetical procedures performed better on most subtests than the group showing deficits in number processing but worse than the other subgroups.

Children with deficits in arithmetic facts retrieval (n = 15, 17%) demonstrated a specific deficit in the time to complete the multiplication table. They also took the longest to complete the written calculation subtest, which also includes multiplication and division. Children with deficits in arithmetic facts retrieval are slower than the other children because they rely on alternative strategies to reconstruct the result of the multiplication tables.

Children classified from the clinicians' team as having trouble in geometrical abilities (n = 7, 8%) performed poorly on the geometry subtest (lower than TD group, LD_notM group, and the group with deficits in arithmetic facts retrieval) and performed significantly lower on the 3D models than that LD_notM group. However, for both subtests, they performed similarly to the group with deficits in number processing. The group with deficits in geometrical abilities was the only group to perform worse than TD on the ANS for dots (p=0.032). In this subtest numbers are represented through dots that might be confusing for children who have trouble in perceiving forms and identifying shapes.

Performance in the Dyscalculia Subtyping Battery							
	Controls		Deficits in mathematics				
Subtests of the DSB	**TD (n=18)**	**LD_notM (n=25)**	**Number (n=10)**	**Arithmetical Procedures (n=15)**	**Arithmetic Facts Retrieval (n=15)**	**Geometrical abilities (n=7)**	**Omnibus Sig (F,p)**
Demographics							
Sex (M:F)	11:7	19:6	4:6	9:6	12:3	4:3	0.3
Handness (R:L)	16:2	23:2	9:1	14:1	14:1	7:0	0.963
Other Diagnoses (Dyslexia: ADHD: Both)	0:0:0	19:3:3	5:4:1	9:0:6	7:2:6	6:1:0	0.288
Age at Math Testing (y)	10.45 (1.63)	11:69 (2.01)	11.68 (2.47)	11.73 (2.49)	12.07 (1.95)	11.97 (1.74)	0.263
Age difference (Math-Neuropsychy)	0.01 (0.03)	0.39 (0.87)	0.38 (0.73)	0.30 (0.64)	0.26 (0.55)	1.20 (1.43)	0.118
Numbers							
Writing numbers in digit (performance in %)	95.56 (7.04)c	95.6 (8.2)c	60 (16.32)*	92.67 (10.32)c	96 (7.37)c	92.86 (11.1)c	23.42, <0.00001
Spelling number in letters (performance in %)	97.78 (4.27)c	93.6 (9.52)c	67 (24.9)*	89.33 (13.35)c	98.67 (3.52)c	98.57 (3.78)c	12.37, <0.00001
Sequences (performance in %)	96.30 (9.13)c,d	85.33 (23.23)c,d	30 (29.2)a,b,e,f	51.11 (27.07)a,b,e,f	85.56 (10.67)c,d	85.71 (11.5)c,d	20.37, <0.00001
Ordering (performance in %)	94 (13.4)c,d	87.69 (18.33)c,d	47.33 (19.8)*	70.04 (19.38)a,b,c	79.2 (15.9)c	74 (18.82)c	11.39, <0.00001
ANS digit (performance in %)	95 (6)*	82 (12)a,c	69 (19)a,b	75 (13)a	81 (8)a	75 (10)a	7.73, <0.00001
ANS dots (performance in %)	81 (11)f	72 (12)	68 (14)	69 (11)	71 (12)	64 (10)a	2.86, <0.02
Arithmetical Procedures							
Additions (performance in %)	98.33 (2.42)c	98.55 (2.78)c,d	86.13 (9.25)*	92.92 (8.6)b,c,e	98.67 (2.22)c,d	96.96 (4.07)c	11.05, <0.00001
Subtractions (performance in %)	97.36 (4.04)c,d	96.2 (4.91)c,d	65.13 (20.09)*	80.5 (15.55)*	93.67 (8.95)c,d	95.36 (5.08)c,d	18.13, <0.00001
Written Calculation (performance in %)	88.19 (13.7)c,d	77.5 (22.46)c,d	31.88 (19.64)a,b,e,f	44.58 (27.53)a,b,e,f	68.33 (20.92)c,d	83.04 (12.87)c,d	14.93, <0.00001
Time Additions (time in seconds)	53.5 (25.54)c,d	80.08 (36.31)c,d	149.3 (59.93)a,b,f	151.47 (84.84)a,b,e,f	95.8 (35.63)d	65.29 (25.39)c,d	10.21, <0.00001
Time Subtractions (time in seconds)	66.78 (32.58)c,d,e	112.48 (66.85)d	183.1 (64.4)a	192.4 (99.6)a,b,f	142.06 (50.55)a	100.57 (41.93)d	8.42, <0.00001
Time Written Calculation (time in seconds)	112 (61.46)e	210.88 (101.17)	171.1 (94.77)	216.13 (189.47)	300.8 (147.6)a	232.57 (101.71)	4.17, 0.002
Arithmetic Facts Retrieval							
Mutiplications (performance in %)	89.65 (25.1)c	85.48 (27.54)c	35.24 (34.16)*	66.90 (25.55)c	82.30 (9.07)c	96.77 (5.78)c	8.99, <0.00001
Time Multiplications (time in seconds)	43.71 (30.14)d,e	71.35 (33.41)e	57.88 (27.56)e	110.29 (69.26)a	142.47 (73.95)a,b,c	77.71 (46.65)	7.78, <0.00001
Geometrical Abilities							
Geometrical Test (performance in %)	82.69 (8.06)c,d,f	81.11 (9.03)c,d,f	61.60 (10.9)a,b,e	66.75 (15.24)a,b,e	82.28 (8.04)c,d,f	63.60 (5.5)a,b,e	12.75, <0.00001
2D Models (performance in %)	86.11 (11.25)	85.5 (13.82)	71.25 (17.72)	74.17 (20.84)	84.17 (15.28)	71.43 (13.91)	2.86, 0.02 *n.s.*
3D Models (performance in %)	88.89 (14)	95.33 (7.63)c,f	76.67 (23.83)b	84.44 (13.31)	84.44 (13.31)	76.19 (16.26)b	3.98, 0.0028

Table 1. Performance in the Dyscalculia Subtyping Battery (DSB).

A table showing the subtests that were used to distinguish the subtypes of mathematical deficit. *diff from everyone; a= diff from typically developing (TD) children; b= diff from children with learning disorder not affecting mathematical abilities (LD_notM); c= diff from the group with deficits in number; d= diff from the group with deficit in arithmetical procedures; e= diff from the group with deficit in arithmetic fact retrieval; f= diff from group with deficit in geometrical abilities. All p-values Bonferroni-corrected for multiple comparisons (p<0.003).

3.3 <u>Group Performance in neuropsychological and academic assessments</u>

Among the 75 children diagnosed by the team of UCSF-DC as having a learning disorder, 68 had dyslexia (dyslexia only n=50 and dyslexia and ADHD n=18). The clinical team assessment and the results from the DSB confirmed that a high percentage of these children were classified as having trouble in mathematics (66%). All but three of these children were able to be classified based on a distinct impairment (n=65). Most of these children were classified related to impairements in arithmetical procedures (23%) and arithmetic facts retrieval (20%). A small part of these children were classified with impairment in geometrical abilities (9%). A significant overlapping between dyslexia and primary dyscalculia was confirmed by the nine children classified with impairments in number processing (14%).

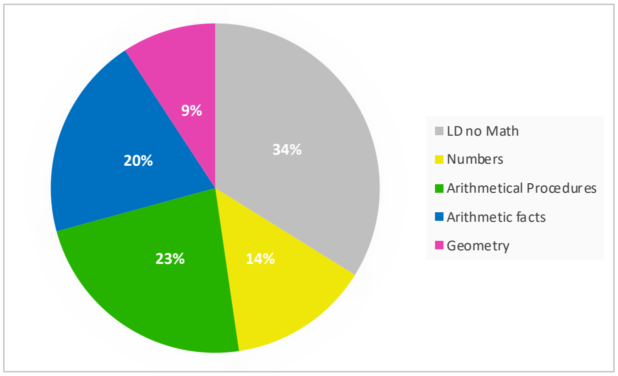

Figure 3. Percentage of children with dyslexia having impairment in mathematics

Despite the small number of children in each group with impairement in mathematics, we nevertheless present results in neuropsychological and academic tests. Among the groups with learning disorders, statistically significant differences were also noted in the performance of WJ Calculations (WJ Calc), non-verbal reasoning (Matrix Reasoning), and judgement of line orientation (JLO) tests (Supplementary Table 2). Specifically, the group with LD_notM had higher scores on the WJ Calc and Matrix Reasoning subtests than the groups showing deficits in number processing and arithmetical procedures. There were also group differences on JLO, but the LD_notM and the group with deficit in arithmetic facts retrieval performed better than the group with a deficit in number processing. On these measures the subgroup showing a deficit in number processing had the lowest performance that ranged from low average to borderline impaired. Groups with impairment in numbers and arithmetical procedures performed significantly lower on Matrix Reasoning. Matrix Reasoning is a non verbal reasoning measure that predicts mathematical performance (Green et al., 2017). However, visual pattern recognition may not be domain-specific to mathematics.

4 Discussion

The UCSF Dyscalculia Subtyping Battery DSB was developed at UCSF-DC to assess strengths and weaknesses in mathematical cognition of children from 2nd to 8th grade. It spans number processing, arithmetical procedures, arithmetic facts retrieval, and geometrical abilities, allowing educational practices and interventions to be tailored to each individual profile. We reported here the initial results of a large cohort of children with known learning disorder (n=75) carefully phenotyped by a group of expert clinicians, and described in detail four exemplar cases illustrating the fine-grained heterogeneity of deficits in these four mathematical domains. The DSB detected these subtypes of mathematical deficits, thus proving to be a promising assessment tool that could be scaled to larger cohorts of children. The following is a discussion of the clinical and educational implications our findings and their relation to current neurocognitive theories of mathematical deficits.

4.1 Detecting and isolating deficits in mathematics

The first main result of our study is that our carefully designed battery can identify children with deficits in mathematics and dissociate them from other learning disorders. Among our cohort, 50 of the 75 participants diagnosed at the UCSF-DC as having a developmental learning disability were shown to have deficits in mathematical processing, rarely identified with standardized diagnostic assessments. Estimates of the prevalence of mathematics learning disabilities vary between 3%–6% and up to 20%, depending on the inclusion criteria, the tests selected, and the benchmark cutoff chosen (Devine et al., 2013; Kaufmann & von Aster, 2012; Shalev & von Aster, 2008). However, these studies limit their inclusion criteria to psychometric cut-offs (and not clinical diagnosis) and most of them have weak statistical power ($N \leq 20$) (Astle & Fletcher-Watson, 2020; Mammarella et al., 2021). It is thus likely that many cases of mathematical defficulties are currently undetected. In addition to that, around 40% of children with reading difficulties have been reported to also have difficulties in learning mathematics (Butterworth, 1999; Dirks et al., 2008; Wilson et al., 2015) making it difficult to disambiguate their profile. What is unique about the DSB is that it was designed from a behavioral neurology approach considering abnormal development based on a confluence of history, known brain-behavior associations, and mathematical development, which increases the sensitivity and specificity of the test. Most of the routinely administered assessments (WJ (Calculation Test, Broad Math, Applied Problems, Fluency), WIAT (Math, Numerical, Problem Solving, Fluency), KTEA (Math Concepts and Applications, Computation, Fluency, Problem Solving), FAM) sometimes miss difficulties in mathematics, specifically if those difficulties are not related to number deficits. For example, 12 children in our cohort were not previously identified as having trouble in mathematics. The impression of the clinician team and the results of the DSB revealed that none of these 12 had deficits in number processing. However, five had deficits in arithmetic facts retrieval, three on arithmetical procedures, and four on geometrical abilities.

The current available mathematical assessments are not designed to distinguish deficits in number processing, arithmetical procedures, arithmetic facts retrieval, geometrical abilities. In particular, geometry is under-assessed and should be included during the assessment of mathematical cognition because it is possible to detect difficulties in this domain even before it is formally taught (in the 10th grade in US), which means that it may be possible to intervene before a child starts to struggle in school. From a very early age, children possess a capacity for geometrical intuition, they are sensitive to shape both in 2D and 3D and can identify some relationships between shapes and forms (Izard & Spelke, 2009; Lourenco & Huttenlocher, 2008; Schwartz & Day, 1979; Slater et al., 1994). Furthermore, difficulties with geometrical abilities can impact a student's ability to utilize mathematical symbols appropriately (e.g. an *x* for multiplication and a + for addition) and read and understand graphs and charts; all of which are important for tests (sciences and math), standardized placement tests, and job skills (Sherard, 1981).

In addition to being able to determine the presence of specific deficits in mathematics, the multi-dimensional nature of the DSB distinguished children with more general learning disorders from those with specific difficulties in mathematics through the use of cognitive neuroscience measures. Deficits in mathematics often present in association with other learning disorders; about 50% of children with deficits in mathematics also have another learning disorder (Morsanyi et al., 2018). Table 1 clearly shows that children with other learning disorders but without deficits in mathematics performed similarly to TD in all subtests, except the ANS for digits (children obtaining low score in the ANS for digits had attention deficits). This demonstrates the ability of the DSB to dissociate deficits in mathematics from learning disorders in general. Overall, we believe that this indicates that the DSB has both better sensitivity (i.e., capturing 12 children undetected by previous assessments) and specificity (i.e., confirming that the 25 children with learning disorder did not have deficit in mathematics).

4.2 Phenotyping developmental dyscalculia

The second main result of our study is that based on previous evidence of the cognitive and neural heterogeneity of mathematical disorders (Skagerlund & Träff, 2016; Träff et al., 2017), it is possible to dissociate primary dyscalculia, characterized by a relatively selective deficit in numerical functioning from other mathematical impairements related to other mathematical skills, such as arithmetic procedures, facts retrieval, and geometrical abilities (Kaufmann et al., 2013).

Researchers have asserted that examining deficits in mathematics with a differential and developmental approach will help to clarify the ambiguity present in research on DD. In this study, we have addressed this need and shown that there is heterogeneity within mathematics learning disorders, and designed a battery to supplant the currently available assessment measures that are not specific. Below we provide clearer descriptions of the severity and pattern of deficits that can aid in differential diagnosis.

Currently, there is no agreed-upon picture of the cognitive profile(s) of children with a specific learning disorder in mathematics. Contrary to many prior studies (Feigenson et al., 2004), recent studies (Astle & Fletcher-Watson, 2020; Mammarella et al., 2021) have countered that there is no evidence for core deficit(s) in mathematics learning disorder. This probably contributes to the fact that, despite the efforts by cognitive researchers, a common, practical definition of dyscalculia has not been agreed upon (Butterworth, 2019) and existing diagnostic assessments seem to still be inadequate.

Our results, using this new developed math cognition battery combined with careful single-case examples, suggest the existence of at least four subtypes of developmental dyscalculia according to the mathematical impairment to which it is related: dyscalculia in number processing, dyscalculia in arithmetical procedures, dyscalculia in arithmetic facts, and dyscalculia in geometry. Future studies using longitudinal data and a larger sample sizes would be necessary to develop a nosological taxonomy with precise diagnostic criteria. Significant group effects were found in all subtests of the math battery, except for the 2D shape reconstruction subtest (where results were at ceiling for all children). We will now briefly discuss potential criteria to label each subtype, and describe the main characteristics and most likely neurocognitive correlates.

Primary DD: Dyscalculia in number processing

Dyscalculia in number processing, or primary dyscalculia, is characterized by impairments that arise from a deficit in connecting a number to the corresponding magnitude (*number sense* for Cohen & Dehaene, 1995, *number module* for Butterworth, 1999). Deficits in number processing inevitably compromises all other mathematical skills: a low score in the whole battery characterized this subgroup.

Deficits were observed in at least 3 of the following tasks: translation of numbers from one code to another (digit, words, pictorial), comparison and ordering of numbers, and filling out sequences of numbers. Sometimes deficits may be observed also in subitizing and more often in ANS, both symbolic (digits) and non-symbolic (dots). We observed no significant difference in the group of children with deficit in number processing in respect to the other groups in the subitizing test. However, these children were often unable to provide a correct estimation of the number of dots when they were more than 5. They often stated that there were 30, 40, 50 or 100 dots even if the maximum number of dots on the screen is 9. In our battery, children with a deficit in number processing performed worse than controls and the other subgroups on the ANS for digits. Although all groups tended to perform better on digits than dots, the children with a deficit in number processing performed only slightly better on digits than dots (difference of one point) whereas the other groups showed greater differences (min difference: 6, max difference: 14). This further highlights the difficulties encountered in the children with a deficit in number processing: it is ubiquitous no matter the presentation. Additionally, previous evidence suggests that children with dyscalculia overall perform worse in non-symbolic (dots) tasks rather than symbolic (digits) ones (Mazzocco et al., 2011; Piazza et al., 2010; Rousselle & Noël, 2007; Schwenk et al., 2017) and highlights the need for the assessment of non-symbolic skills without the confound of symbolic representations. Overall, symbolic (digits) number skills seem to develop independently from the ANS (Lyons et al., 2018; Matejko & Ansari, 2016) and while symbolic (digits) number abilities predict non-symbolic (dots) number processing, there is weak or non-significant evidence of the reverse (Lyons et al., 2018). Furthermore, a recent study suggests a correlation between ANS and executive function mechanisms highlighting that a low score in the ANS is insufficient to identify dyscalculia (Wilkey et al., 2020).

Given the positive relationship between numerical skills and visuospatial skills (Sella et al., 2016), it is not surprising that no significant difference was observed between children with a deficit in number processing and children with a deficit in geometrical abilities in the geometrical subtests.

Primary dyscalculia is usually the easier one to detect even with the current diagnostic tests and is the one that has received the most research attention. Contrary to what might be expected, in our sample only 21% (10/47) of the children were diagnosed with deficits in number processing.

This means that only a small part of children who struggle in mathematics have a primary developmental dyscalculia.

Children with deficits in number processing show impairments in many mathematical domains, including calculation skills and visuospatial abilities. The group with deficits in number processing had a performance on the WJ calculation test ranging from borderline to impaired, significantly lower than any other group. In the Beery, children with a deficit in number processing as well children with a deficit in geometrical abilities obtained the lowest scores. Furthermore, children with deficit in number processing demonstrated difficulties in several tests that require visual reasoning and judgements but had spared verbal knowledge. This suggests that a cognitive mechanism of higher-order non-verbal reasoning and attention may underlie the difficulties observed in this subgroup.

Dyscalculia in arithmetical procedures

Deficits in arithmetical procedures are characterized by a focal impairment when applying arithmetical procedures. While not showing deficits with numerical skills, children with deficits in arithmetical procedures show difficulties in understanding, applying, and reproducing mental and/or written calculation.

In the DSB, these children performed significantly differently from the other children both in terms of accuracy and speed in calculations. Errors include treating subtraction as a commutative operation (e.g., $5-3 \neq 3-5$) and inverting the order of digits (e.g., $30-11=21$ because $3-1=2$ and $1-0=1$), while slow processing was due to the adoption of ineffective *counting strategies,* often supported by fingers or drawing.

Children with deficits in arithmetical procedures obtained lower scores than LD children without mathematical deficits on WJ Calculations. This is expected because most of the problems in this test require calculation abilities. These children also obtained lower scores in non-verbal reasoning (Matrix Reasoning) and visual divided attention (Children's Colored Trails). In the visual divided attention, they had a performance level similar to the group with deficits in number processing.

Unexpectedly, children with a deficit in arithmetical procedures performed poorly on some of the geometry subtests. A common denominator for this pattern of difficulties could be working memory, which has been previously suggested (Berg, 2008; DeStefano & LeFevre, 2004).

However, in our study, this group did not show group differences on working memory measures. Another possibility is that this group may have some visuospatial difficulties that make it difficult for them to keep numbers aligned during arithmetical procedures and therefore they make errors of alignment (Russell & Ginsburg, 1984). We do not have evidence to support this possibility in our data (only a few errors are due to misalignment of numbers in columns). However, more than 50% of the children with deficits in arithmetical procedures (8 out of 15) erroneously calculated the difference between two numbers, subtracting the smaller digit from the larger without respecting the order of the subtraction with regard to the whole number (e.g., in 90 – 47, they subtract 0 from 7 and 4 from 9). This error could be associated with deficits in the spatial representation of quantitive information (Raghubar et al., 2009) because "direction" of the operation is incorrectly read. They may also be rigidly applying one rule, "subtract the smaller number from the larger," at the expense of other rules, which may suggest difficulties ranking competing choices.

It is also possible that when we perform written calculations we are always dividing our attention. If someone has proficient calculation skills and knowledge, they may be dividing their attention between the motor program of writing and the estimation of the correct result in order to check their work simultaneously. However, a child who has not mastered calculations may instead divide their attention between the motor program of writing and searching their mind for the appropriate calculation rules/algorithms for those numbers. If this is the case, then interventions for this type of deficit may focus on alternate strategies that reduce the burden of divided attention.

Dyscalculia in arithmetic facts

Deficits in arithmetic facts are characterized by impairments in arithmetic facts retrieval. Children with deficits in arithmetic facts do not show difficulties on numbers subtests and they are able to perform calculations. Yet, they struggle in recalling the result of operations that should have been learned by rote memorization, such as multiplication facts. The deficit in recalling multiplication tables is paired with difficulties with written multi-step multiplications and divisions.

In the DSB, children with deficit in arithmetic facts took the longest to complete the multiplication table subtest and the written calculation subtest. Critically, it is important to evaluate not only the overall performance but also the response time: slow responses are likely associated with compensatory strategies that are only partially effective. For instance, complex multiplication can be solved by relying on the lengthy mental strategy of repeated addition (e.g., 8×4 is broken down into 8+8=16 and then 16+16=32). Children with deficits in arithmetic facts generally performed well in addition and subtraction tasks. Subtraction and multiplication are associated to distinct neural systems for numerosity and language (Prado et al., 2011).

Although fluency in reading and arithmetic are referred to as distinct abilities, significant correlations were found among them (Balhinez & Shaul, 2019; Singer & Strasser, 2017). Due to the similarities between the first stages of development of reading and mathematical fluency it is likely that the same brain and cognitive mechanisms are involved in both domains (Qin et al., 2014). As our particular cohort was recruited based on symptoms of dyslexia, it is difficult for us to parse apart these differences. Future studies that use the DSB to evaluate performance of children with dyslexia compared to dyscalculia could help to address this question.

Like the group with no deficits in mathematics, children with deficits in arithmetic facts performed better than the other groups on untimed visual matching (Judgement of Line Orientation (JLO)). Interestingly, similar results were found in the DSB where these children obtained higher scores than the other groups with math difficulties on geometry subtests.

The relation between arithmetic facts retrieval and language abilities (De Smedt & Boets, 2010) might explain this result. Perhaps these children developed better visuospatial skills either in response to or because of their language difficulties, i.e. balanced resources across the brain can lead to increased functionality in brain regions opposite of regions with diminished functionality (Qin et al., 2014). Future research that examines the developmental trajectory of language and visuospatial skills in children at risk for dyscalculia will help to understand these findings.

Dyscalculia in Geometry

Deficits in Geometry are characterized by impairments in geometrical abilities. Children with deficits in geometry show deficits in the nonverbal representations of mathematical information. They might struggle to process distances and directions, match shapes, recognize geometrical transformation (e.g., symmetries, rotations), and mentally reconstruct a 3D model from a 2D shape.

In the DSB, children with deficits in geometrical abilities had the lowest scores on the geometry subtest and performed significantly lower on the 3D model test (lower than TD, group with no deficits in math, and group with deficits in arithmetic facts). In the computerized test (Izard & Spelke, 2009), they made mistakes in solving problems related to geometrical transformations (symmetries, rotations and translations), in distinguishing distances and in identifying characteristics of geometrical figures.

Remarkably, the group with deficit in geometrical abilities was the only one to perform worse than TD on the ANS for dots. There is a positive correlation between numerical skills and visuospatial skills; it is not surprising that these subjects struggled in connecting numbers with visual objects (Sella et al., 2016).

Some recent research suggests that spatial ability predicts performance in mathematics (Tosto et al., 2014) but it is still unclear which mathematical skills are involved. Children with deficits in geometry performed well in calculation tasks both in the WJ test and in the DSB. However, our group of children with deficits in geometry is not large enough to make conclusions on this point and further research is necessary to corroborate our data. Even if not significantly different, their scores on the visuospatial processing (Beery) were the lowest.

4.3 Math assessments capable of distinguishing subtypes of mathematical deficits

To our knowledge, there are currently only two standardized batteries able to identify mathematical deficits subtypes. The Feifer Assessment of Mathematics – FAM (Feifer, 2016) targets the distinction between a procedural, a verbal, and semantic type of mathematical deficits, while the Detecting Difficulties in Mathematics battery – DeDiMa (Karagiannakis et al., 2014) allows classification according to 4 domains: numerical, visuo-spatial, memory, and reasoning skills. Although taking an approach similar to ours, there are important differences between these batteries and the DSB. In FAM, for instance, a problem routed in number sense weakness would affect both the procedural and the semantic score. Therefore, it does not unify the weakness in the number sense and possibly detracts from a common mathematical learning profile that underlies that particular subtype of DD. Similarly, DeDiMa does not allow identification of specific didactical issues and covers only a restricted age range (children of 10-12). Both the DSB and DeDiMa aim at identifying specific difficulties in mathematics with the explicit goal of designing appropriate didactical interventions. Yet we take a major step forward by considering more scholastic levels: our battery is designed for 2nd grade up to 8th grade, while DeDiMa is designed for 5th and 6th grades. Therefore, the battery we designed is more sensitive to identifying specific mathematical learning difficulties in a larger age range and excludes the effects of didactical factors.

A growing body of neuroimaging evidence corroborates the interpretation of dyscalculia as a multidomain developmental disorder with diverse neurological underpinnings. For instance, so-called pure dyscalculia (a label assigned to cases that would fit our definition of dyscalculia in numbers) has been associated with abnormalities in the left IPS (Ansari, 2008; Isaacs et al., 2001; Landerl et al., 2004). Moreover, arithmetic problem-solving difficulties have been associated with aberrant responses (hyper-activity and hyper-connectivity) in a number of posterior brain areas suggesting a critical role of parietal circuits in deficits related to arithmetical procedures (Rosenberg-Lee et al., 2015). Conversely, the medial temporal lobe, specifically the left hippocampus, have been implicated in arithmetic facts retrieval (Cho et al., 2012; De Smedt et al., 2011). Finally, posterior inferior-temporal cortex (including the fusiform gyrus) and the posterior parietal cortex have been linked with geometry problem solving (Amalric & Dehaene, 2019), but further studies might aim at disentailing the specific neural correlates associated with visuo-perceptual, visuo-spatial, and visuo-constructional deficits.

While acknowledging the limitations still present in our approach (see below), we believe that the DSB paves the way to clinically meaningful, neurocognitively informed, diagnoses and models.

4.4 Implications for treatments and interventions

Literature indicates that students with math difficulties continue to struggle with mathematics in later grades making it a priority to identify appropriate educational interventions as early as possible to prevent this problem (Nelson & Powell, 2017).

First of all, educators and clinicians need to tease apart cases stemming from neurodevelopmental disabilities from those resulting from ineffective instruction (Heyd-Metzuyanim, 2013) and adapt their educational practices as needed (Karagiannakis et al., 2014). For example, a widely adopted strategy to teach fractions is that of using visual representations (e.g., boxes, rectangles). This approach can lead to incorrect conceptions (e.g., ¾+¾ = 6/8 is incorrect but looks plausible if two pies are used to represent it). Being taught to reason about fractions this way, children might systematically fail in mathematical problems involving fractions even in absences of dyscalculia (Ball, 1993).

Second, intervention programs for dyscalculia should be tailored to the individual neurocognitive profile highlighted by proper in-depth assessment. For instance, a study suggests to incorporate numerical symbols into informal play activities at an earlier age to promote the numerical development and mediate between informal and formal mathematical competences (Merkley & Ansari, 2016). However, a child with deficits in calculations would not benefit from a training designed to strengthen the connection between the concept of magnitude and the symbolic representation of number (e.g., exercises on the number line) (Woods et al., 2018). Rather, calculation skills might be boosted by intervention directly addressing mental strategies (e.g., adding 9 to a number means adding 10 and subtracting 1). Ultimately, our goal is to develop specific didactical interventions for each of these four subtypes of dyscalculia.

4.5 Limits and future perspectives

The DSB has not been standardized yet and, as described above, the current criteria rely on clinical, qualitative evaluations. We are currently administering a second version, slightly modified based on the results here presented, to a large independent sample to enable proper standardization and identify a-priori cut-off scores. Nevertheless, the qualitative classification here presented illustrates the potential of DSB as a tool allowing phenotypical subtyping of DD (Supplemental Table 3: the preliminary diagnostic decision guide).

Although the DSB was successful at confirming subtypes of deficits in mathematics, some of the subsections appear to corroborate the presence of dyscalculia, yet do not provide additional information on the specific subtype. For instance, subitizing and the ANS for digits appear to simply differentiate typically developing children from children with learning differences.

Three subjects showed a pattern of errors so diverse as to preclude labeling in a specific subtype. As for all neurocognitive continuums that can be broken down into clinically meaningful phenotypes, there will likely always be dyscalculia cases that cannot be ascribed to a specific label (Peters & Ansari, 2019). However, future studies might help to elucidate the unique cognitive and neural correlates of these mixed cases.

We have identified four subtypes of dyscalculia: DD in number processing, DD in arithmetical procedures, DD in arithmetic facts retrieval and DD in geometry. This is a preliminary classification of DD. Further distinctions can be added. For example, transcoding numbers and understanding the magnitude of numbers are skills associated to different neurocognitive profiles (Haase et al., 2014) suggesting that DD in number processing might be subdivided in two distinct groups.

Finally, the cohort of children we tested (n=93) is relatively small, preventing more advanced statistical comparisons of the distinct subtypes of DD identified. It should also be noted that these participants had a concomitant diagnosis of dyslexia and/or ADHD, and further studies are needed to allow generalization of the current findings.

Aiming at large-scale standardization, future work will include two versions of the DSB presented: a similar yet improved long form allowing in-depth evaluation of strengths and weaknesses, as well as a shorter form to be used as screening tool.

5 Conclusion

We presented a novel approach to phenotyping subtypes of mathematical deficits: the DSB, encompassing numerical processing skills, arithmetical procedure, arithmetic facts retrieval, and geometrical abilities. Tested in a large cohort of children from 2nd to 8th grade, our battery can help identification of individual strength and weakness in mathematical cognition and isolate primary dyscalculia, dissociate impairments in number processing, from other three distinct mathematical deficits that could define 3 supplementary subtypes of dyscalculia: DD in arithmetical procedures, DD in arithmetic facts retrieval, DD in geometry. Our preliminary findings pave the way to a personalized medicine approach to the heterogeneous and multifaced disorder that is developmental dyscalculia.

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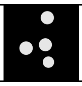
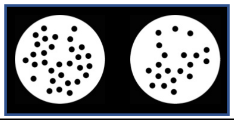
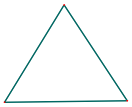
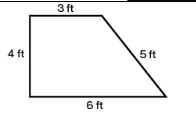
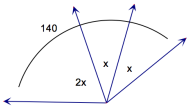
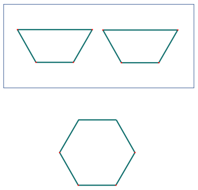
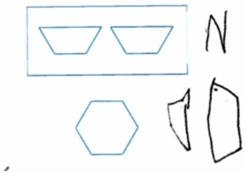
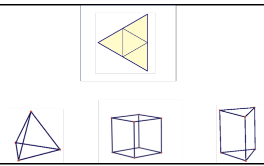
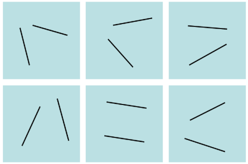
Supplementary Table 1: Description of the subtests in the DSB

Domain	Task	Description	Example	Incorrect response example
num	1	Write numbers in digits	$Eighty - nine \longrightarrow 89$	$Eighty - nine \longrightarrow 98$
num	2	Write numbers in words	$69 \longrightarrow Sixty - nine$	$69 \longrightarrow Sixty - six$
num/ calc	3	Fill out sequences	$Fill\ out\!:\ \ 10\ \ 8\ \ \_\ \ 4\ \ 2\ \ \_$	$10\ \ 8\ \ 7\ \ 4\ \ 2\ \ 1$
num	4	Comparing numbers	$Circle\ the\ bigger\ number\!:\ \ 35\ \ \ 53$	$\textcircled{35}\ \ \ 53$
num	5	Ordering Numbers	$Order\ from\ least\ to\ greatest\!:$ $100\ \ 101\ \ 110\ \ 132\ \ 92\ \ 103$	$92\ \ 100\ \ 103.\ \ 110\ \ 101\ \ 132$
num	6	Estimation	$51 + 69 =$	$51 + 69 = 100$
num	7	Subitizing		$There\ are\ 5\ dots$
num	8	Approximate number system		$There\ are\ more\ dots\ on\ the\ right$
calc	9	Mental additions	$Find\ the\ sum\!:\ 19 + 5$	$19 + 5 = 23$
calc	10	Mental subtractions	$Find\ the\ difference\!:\ 12 - 5$	$12 - 5 = 8$
fact	11	Mental Multiplications	$Find\ the\ product\!:\ 6 \times 9$	$6 \times 9 = 56$
calc	12	Multiples of numbers	$List\ out\ the\ multiples\ of\ 4\ until\ the\ number\ 32$	$4, 8, 12, 17, 21, 25, 29, 32$
calc / fact	13	Written calculation	$33 + 26 =\quad 90 - 47 =\quad 16 \times 72 =$ $108 \div 9 =$	$90 - 47 = 57$
num / calc / fact / did	14	Find the missing sign in the expression	$(1 + 8)\ ?\ 5 = 45$	$? = +$
num / calc / fact /did	15	Find the missing number in the expression	$88 = ? + 80$	$88 = 18 + 80$
num / calc / fact / did	16	True/False	$Is\ it\ true\ or\ false?\ \ 4 \times (2 + 10) = 18$	$True$
num / did	17	Equivalent Fractions	$\frac{1}{2} = \frac{?}{6}$	$\frac{1}{2} = \frac{2}{6}$
num / did	18	Percentage	$0.5 = \frac{\quad}{100} = \_\%$	$0.5 = \frac{5}{100} = 5\,\%$
num / calc / fact / did	19	Word problems	$How\ do\ you\ share\ 24\ cupcakes\ equally\ between$	$2\ cupcakes\ for\ child$
did	20	Simplifying expressions	$9 + 8 \div (-4) - 2$	$9 + 8 \div (-4) - 2 = 2$
did	21	Solving equations	$9 - 3x = 1 + x$	$x = 5$
geometry	22	Name the figure		$Rectangle$
geometry / did	23	Perimeter and Area		$Perimeter = 17\ ft$ $Area = 360\ ft^2$
did	24	Modeling		$x = 70$
geometry	25	2D shape reconstruction	$Can\ you\ make\ the\ shape\ using\ the\ two\ pieces\ in$ 	No 
geometry	26	From 2D shape to 3D model	$Which\ 3D\ model\ does\ the\ figure\ in\ the\ box\ crea$	$Model\ number\ 3$

				
geometry	27	Geometry-Test	*Which figure does not belong to the set?* 	*Figure # 6*

Supplementary Table 1: Description of the subtests in the DSB
Did = These subtests are used to eliminate or confirm didactical issues.

Num = These subtests are used to identify deficits in number processing

Calc = These subtests are used to identify deficits in arithmetical procedures

Fact = These subtests are used to identify deficits in arithmetic facts retrieval

Geometry= These subtests are used to identify deficits in geometry

Supplementary Table 2: Demographic and neuropsychological performance

	LD_notM	#	Numb	#	Arith Proc	#	Arith Facts	#	Geom	#	Sign. (p)
	Ave(std)		Ave(std)		Ave(std)		Ave(std)		Ave(std)		
General Cognitive and Academic											
WJ-IV Oral Vocabulary	60.8 (25.7)	24	35.8 (38.1)	7	47.6 (21.3)	14	52.7 (21.7)	15	61.7 (25.9)	7	0.16
Receptive Vocabulary -ROWPVT	72 (25.2)	23	57.4 (34.8)	7	64.7 (23.2)	14	77.8 (16.1)	14	85.5 (16.2)	7	0.14
WASI Matrix Reasoning	72 (27.3)c,d	23	24.4 (30.4)b	7	34.7 (23.3)b	14	55.1 (22.8)	15	56.1 (23.4)	7	**<0.001**
WJ-IV Calculations	37.4 (26.7)c,d	24	3.6 (3.4)b	7	13.2 (13.6)b	14	21.5 (19.5)	15	28.7 (10.4)	7	**0.001**
Reading											
(TOWRE-2) Sight Words	24.6 (24.4)	24	4.16 (6.7)	7	15.2 (13.5)	14	25.5 (23.5)	15	18.9 (25.4)	7	0.16
(TOWRE-2) Pseudowords	19.1 (18.8)	24	3.4 (4.5)	7	15.5 (10.6)	14	19.8 (22.8)	15	14.9 (19.8)	7	0.3
Processing Speed											
WISC-IVi Symbol Search	55.7 (27.8)	22	21.8 (12.1)	6	29.5 (28.0)	13	42.5 (26.3)	15	44.6 (17.5)	7	0.016
WISC-IVi Coding	25.7 (27.9)	22	11.4 (14.1)	7	14.9 (14.5)	13	23.3 (13.6)	15	29.1 (28.0)	7	0.344
CCT 1 (timed number seq)	41.5 (32.3)	23	34.5 (34.0)	7	30.7 (29.2)	12	38.3 (26.9)	15	32.4 (27.6)	7	0.865
Visuospatial Processing											
Judgement of Line Orientation	50.9 (34.2)c	23	6.4 (15.3)b,e	8	22.9 (30.1)	12	46.6 (22.2)c	14	36.7 (24.6)	7	**0.002**
Beery VMI	40.1 (25.5)	23	15.1 (10.6)	8	22 (26.3)	13	32.3 (25.1)	15	12.0 (6.1)	7	0.015
Memory											
Short-term Verbal - WISC-IVi- Digits Forward	44.8 (30.6)	23	20.9 (16.2)	7	39.4 (25.3)	14	38.9 (26.5)	15	31.0 (27.2)	7	0.322
Long-term Verbal - CVLT LDFR	53.7 (31.7)	22	33.2 (23.8)	6	41.7 (31.2)	14	58.9 (33.6)	14	67.7 (19.5)	7	0.18
Short-term Visual - WISC-IVi- Blocks Forward	43.8 (28.7)	19	31.3 (27.9)	7	38.5 (31.3)	11	37.4 (27.4)	14	40.1 (20.8)	7	0.886
Long-term Visual - Rey-Osterrieth Figure 3' Delay	39.2 (34.81)	22	12.4 (20.8)	7	14.0 (16.9)	14	47.8 (36.7)	15	13.0 (18.3)	7	0.005
Executive Functions											
Flanker	39.5 (27.2)	16	24.6 (32.1)	5	35.5 (30.9)	11	50.2 (28.7)	12	28.8 (26.1)	4	0.456
Verbal Working Memory- WISC-LVi- Digits Backward	39 (23.1)	23	10.6 (7.5)	7	31.7 (28.5)	14	42.1 (21.1)	15	35.6 (30.6)	7	0.06
Visual Working Memory - WISC-LVi- Blocks Backward	49.4 (32.0)	18	22.6 (15.6)	7	29.2 (23.8)	11	30.4 (24.9)	14	49.0 (31.7)	7	0.082
CCT 2 (timed number seq/switch)	37.9 (20.3)	23	12 (12.3)	7	16.5 (18.8)	12	26.0 (20.9)	15	25.7 (17.4)	7	0.008
DKEFS Design Fluency -filled	56.1 (22.4)	19	49 (32.9)	6	47.6 (24.2)	11	66.3 (25.6)	15	43.0 (43.6)	4	0.331

Supplementary Table 2. Demographic and neuropsychological performance in percentile score based on ANOVA. b= diff from LD_notM, no deficits in mathematics; c= diff from the group with deficits in number; d= diff from the group with deficit in arithmetical procedures; e= diff from the group with deficit in arithmetic fact retrieval. All p-values Bonferroni-corrected for multiple comparisons (p<0.003).

Supplementary Table 3: Preliminary diagnostic decision guide

DD in number processing	
Inclusion: at least one of criteria 1-3 must be answered positively	
1.	A severe deficit in writing numbers in digits, e.g. performance ≤ 40% correct
2.	A severe deficit in ordering and sequencing numbers
3.	A systemic pattern of weakness (e.g. performance ≤ 70% correct) in at least three different number tasks, e.g. converting between numbers, words, digits; number comparisons; ordering; sequencing; and approximate number system.
Exclusion: criteria A. must be answered negatively	
A.	Didactical issues (e.g., the child has not learned a mathematical concept yet)
DD in arithmetical procedures	
Inclusion: at least one of criteria 1-3 must be answered positively:	
1.	A severe deficit in at least one of three calculation tasks (mental addition, mental subtraction, or written multi-step calculations)
2.	A systemic pattern of weakness (e.g. average performance < 70% correct) on calculation tasks
3.	Accurate but slow calculations (e.g. average performance above 70% correct but average time to complete each task is > two minutes (up to three minutes for children in the 4th grade or lower).
Exclusion: criteria A. and B. must be answered negatively	
A.	Didactical issues (e.g., the child has not learned a mathematical concept yet)
B.	Deficit in problems involving numbers (e.g., converting between numbers, words, digits; number comparisons; ordering; sequencing; and approximate number system)
DD in arithmetic facts retrieval	
Inclusion: Both criteria (1 & 2) with at least one of the conditions "a" or "b" must be answered positively:	
1a.	A deficit in recalling the multiplication table, e.g. performance ≤ 70% correct
1b.	Accurate but slow multiplication performance, e.g. time to complete ≥ two minutes
2a.	A severe deficit in performance on written multi-step calculations
2b.	Written multi-step calculations take more than two minutes for children in 5th grade or above and greater than four minutes for children up to 4th grade.
Exclusion: criteria A. and B. must be answered negatively	
A.	Didactical issues (e.g., the child has not learned a mathematical concept yet)
B.	Deficit in problems involving numbers (e.g., converting between numbers, words, digits; number comparisons; ordering; sequencing; and approximate number system)
DD in geometry	

<table>
<tr><td colspan="2">Inclusion: Both criteria (1 & 2) with at least one of the conditions “a” or “b” must be answered positively:</td></tr>
<tr><td>1.</td><td>A deficit in the computerized geometrical test e.g. performance < 65% correct for children up to 4th grade and < 75% for children in 5th grade or above</td></tr>
<tr><td>2a.</td><td>A systemic pattern of weakness (e.g average performance < 70% correct on the three geometrical tasks (2D model, 3D model and computerized test)</td></tr>
<tr><td>2b.</td><td>Average time to complete each geometrical task is more than two minutes.</td></tr>
<tr><td colspan="2">Exclusion: criteria A. and B. must be answered negatively.
<i>N.b., the Geometry DD may be present with other types of DD as long as it is the predominant area of weakness.</i></td></tr>
<tr><td>A.</td><td>Didactical issues (e.g., the child has not learned a mathematical concept yet)</td></tr>
<tr><td>B.</td><td>Visual impairments (e.g., convergence problems)</td></tr>
</table>

Supplementary Table 3: Preliminary diagnostic decision guide

In this tables are described the preliminary inclusion and exclusion criteria for the diagnosis of deficits in mathematics based on the DSB